DESIGN OF A TWO-SPAN ARCH BRIDGE WITH ELASTIC PIER USING A METHOD OF MOMENT DISTRIBUTION FOR THE ANALYSIS

A THESIS

Submitted in partial fulfillment
of the requirements for the Degree
of Master of Science

by

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Georgia School of Technology
Atlanta, Georgia
1941

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Approved:

Date Approved by Chairman 2 June 1941

ACKNOWLEDGMENTS

The writer wishes to express here to Dr. F. C. Snow his appreciation for the very helpful advice given during the prosecution of this work. He also wishes to thank Professor F. Bogle and Professor M. H. Bilyk for their valuable assistance in making soundings and surveys.

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STANDARD NOTATION

A_s Cross-sectional area of reinforcement steel

b Width of beam

b For elastic pier analysis the distance from crown to elastic center of arch

C Distance from centroid to outermost fibre, for use in flexure formula

d Effective depth, distance from compression side of concrete to center of reinforcement in tension

d' Distance from C.G. of tensile steel to tension side of beam

Elastic weight of small segment of arch axis $= \frac{ds}{I}$

ds Length along arc (or cord) of small division of arch axis

dx Horizontal projection of small division of arch axis

DL Dead load

E Effective width (or a width over which a load of one wheel is considered distributed).

E_c Modulus of elasticity of concrete

E_m Effective width over which a wheel load is considered distributed for computation of bending moment in floor slab

E_s Effective width over which a wheel load is considered distributed for computation of shear in floor slab

E_s Modulus of elasticity of steel

F Stiffness factor for moment distribution $= \frac{EI}{L}$ or for members having the same E it may be written $\frac{I}{L}$

f_c Compressive unit stress in concrete

f_s Tensile unit stress in reinforcement

$G_{1,2}$ etc. Point on the arch where columns rest and hence where the live load is applied

h Thickness of arch rib

H_0 Horizontal reaction of arch at the springing

I Second moment of area or moment of inertia

IL Impact load

j Ratio of lever arm of resisting couple to effective depth

k Ratio of depth of neutral axis to effective depth

KP Thousand pounds

l Span

L Span

LL Live load

m Distance from left springing line to any point on the arch axis, used chiefly in calculation of influence ordinates

M Bending moment

MF Fixed end moment

M_0 Moment reaction of arch at the springing

n $\frac{E_s}{E_c}$

O Perimeter of bar

P Concentrated load

ϕ Inclination of arch axis to the horizontal

ϕ Symbol for round reinforcing bar

$\not\phi$ Symbol for square reinforcing bar

P_m Load per foot width of slab used in computing bending stresses in the floor slab

P_v Load per foot width of slab used in computing shear stresses in the slab

s Clear span, between edges of support beams or girders

T Width of tire in feet, on truck taken as one inch for each 1000 lb of wheel load

u Bond stress per unit area of bar surface

v Unit shearing stress

V Total shear

V_0 Vertical reaction of arch at the springing

x Horizontal distance from left end of arch axis at skewback

X_0 Eccentricity of column load

y Vertical distance from horizontal line through arch axis at skewback

Special notation for use in the elastic pier analysis is given on page

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CHAPTER I

STATEMENT OF THE PROBLEM

The purpose of this thesis is the design, or to be more precise, the calculation of the stresses in a two-span arch bridge having an elastic pier between the two arch ribs. The superstructure, especially the floor girders, have been analyzed carefully. Two assumptions as to the distribution of the floor load over the arch rib have been followed through, and the results in dead load shear and moment diagrams have been given. The arch ribs have been analyzed, using two assumptions as to the fixity of the center support. First, the assumption that both end supports of each arch were fixed results in the usual fixed right arch analysis. The second assumption is that the center pier was an elastic structural member capable of enough distorsion under load to appreciably affect the stresses in the arch ribs. The results of the two analyses may be seen from the stress tabulations, the one on page 114 the other on page 147.

The theory used in the elastic pier analysis is that of Mr. Alexander Hrennikoff and was taken from a paper published by him in the transactions of the American Society of Civil Engineers.[1] The writer has applied the theory thus presented to the principle of distributing a unit thrust and a unit couple at the common junction of arch ribs and pier, a principle due, it seems, to Professor Hardy Cross.[2]

In recent years considerable interest has been displayed by many engineers in the subject of analyzing arches on elastic piers, and while no new theory is presented here, the writer believes that the arrangement of the calculations and the tabulation of data as used in this thesis will be found satisfactory when the construction of influence lines is desired.

The purpose of the writer of this thesis might have been achieved using data entirely assumed, but it was believed that a better tone would be given the entire work by making a design which would fit some spot in need of a new bridge. After some investigation, the crossing of Howell Mill Road over Peachtree Creek was selected as one adapted to the purpose of the thesis. The crossing here is quite high and is one where an arch bridge could be placed (not every bridge site is suited to the arch). It is not claimed

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[1] Alexander Hrennikoff, "Analysis of Multiple Arches," Transactions of American Society of Civil Engineers No. 101 pp. 388-421.

[2] H. Cross and N. D. Morgan, Continuous Frames of Reinforced Concrete (New York: John Wiley & Sons, Inc.), pp. 338.

that the arch would prove to be the most economical type of bridge for this site; probably it would not. This design is not intended as an economic study.

The only knowledge regarding the nature of the sub soil available to the writer was obtained by himself with the aid of some friends, using a heavy iron bar about eight feet long and a sledge hammer. This was driven into the soil at many points along both sides of the stream and gave definite indications of rock at about four feet below the surface of the stream. This was assumed to continue at the same level in both directions as far as the abutments. In the absence of contractor's equipment for making a professional sounding job, there was little else that could be done. It may be remarked here that the present bridge was erected in 1912, and inquiry disclosed the fact that records were available only as far back as 1913 and hence not obtainable for this site.

Partly because of the lack of definite information regarding the foundation conditions, it was not thought worth while to give an elaborate foundation and wing wall design; however, the writer has presented what he believes to be a satisfactory design, one which would make use of the material now existing in the present piers.

The writer believes that his choice of members and proportions would prove to be satisfactory if used in the erection of the bridge described in this thesis. He has given the principal dimensions and sizes and has outlined the more important details, but it has not been his intention to furnish a complete set of working drawings.

GENERAL SPECIFICATIONS

This bridge is designed to accommodate three lanes of traffic. It is to be a Class AA bridge and, hence, capable of safely carrying Class H-20 loading. A description of this loading is given on a following sheet.

The structural computations are based upon concrete having an ultimate strength of 3000 lb per sq in and reinforcing steel stressed to not over 18000 lb per sq in.

The arch ribs and superstructure are designed for stresses caused by dead load, live load, and changes of temperature. The temperature stresses are computed on the assumption of a rise of 30° F and a drop of 40° F. Wind stresses in the arch ribs are not considered because of the large ratio of width to span of the arch that this bridge has. This practice is as recommended in the "Final Report of the Special Committee on Concrete and Reinforced Concrete Arches," Paper No. 1922 of The American Society of Civil Engineers. Because the superstructure is joined firmly to the arch at the crown girders and, hence, cannot deflect without deflecting the arch rib, wind stresses in the superstructure are not computed.

The design follows, in general, the specifications of the Georgia State Highway Department and the recommendations of the American Concrete Institute.

DESCRIPTION OF CLASS H-20 LOADING

Class H-20 loading consists of one truck of 20 tons gross weight followed by or preceded by or both followed and preceded by a line of trucks of indefinite length, each of the following or preceding trucks having a gross weight of 15 tons.

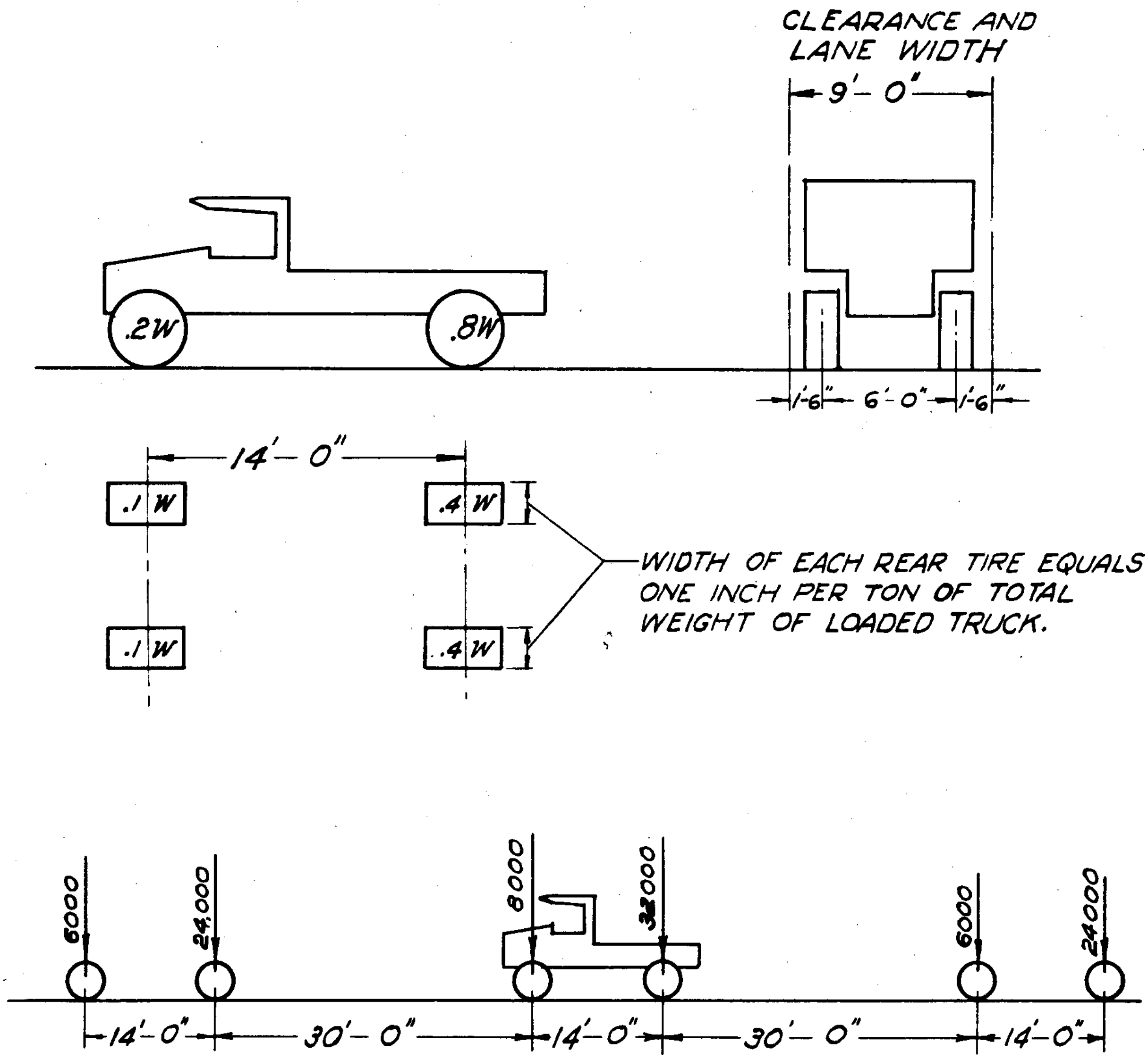


FIG 1

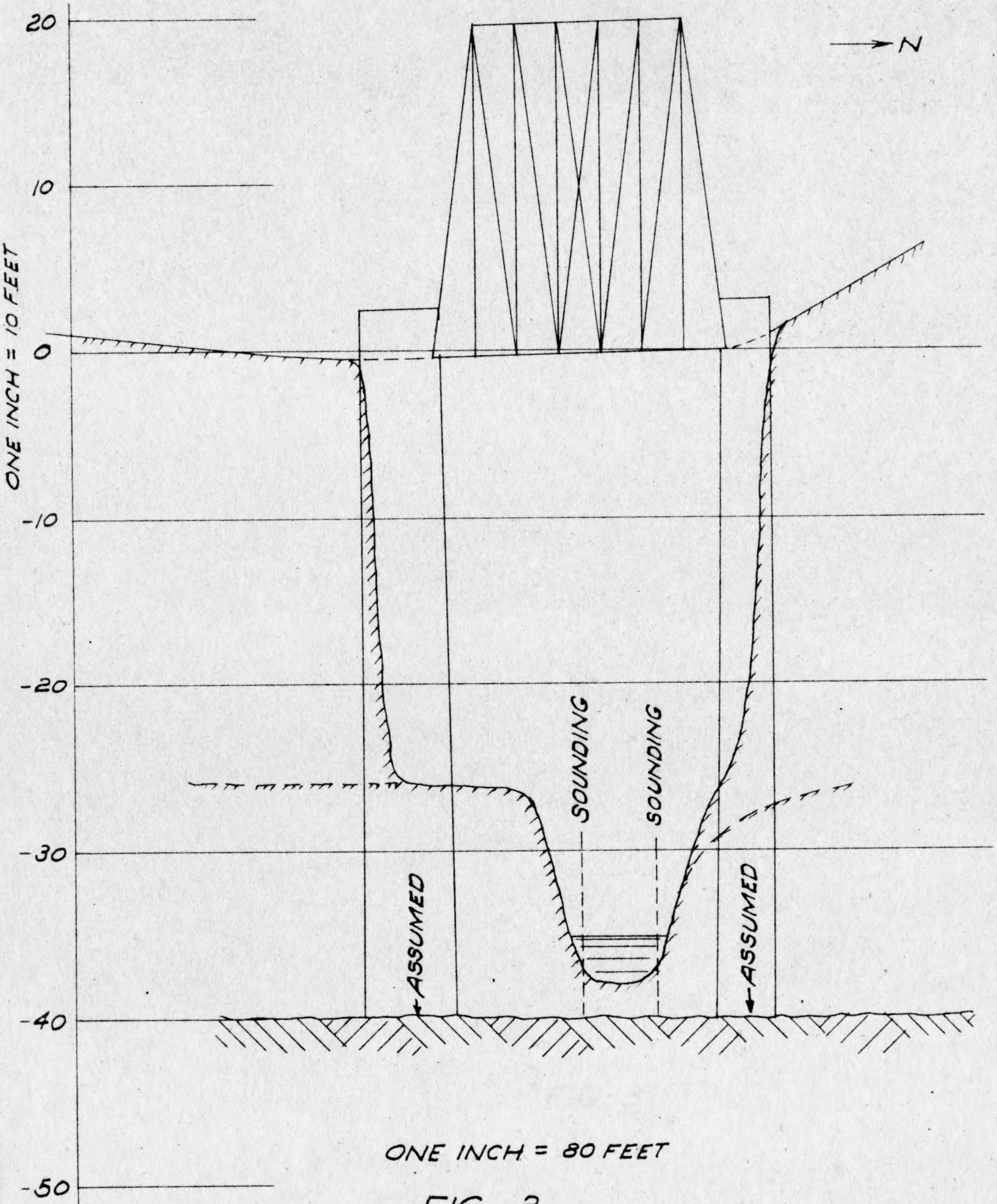


FIG 2

PROFILE DATA

2

PLAN AND ELEVATION
BRIDGE OVER PEACHTREE CREE
SCALE 1/8" = 1'-0"

FIG 3

MAY 15 1941

DRAWN BY F

CHAPTER II

DESIGN OF FLOOR SLAB

By reference to Figure 3 it may be seen that the slab is supported by girders placed at right angles to the direction of the road way. The four girders near the center of each arch rest directly on the arch rib; the three girders at each end of each arch are supported by columns. Some fixity of restraint is offered at each girder. The greatest positive moment will occur somewhere in the first floor span of each arch, and the greatest negative moment will occur over the first girder on the arch proper. It is considered here as being quite accurate enough to compute these moments on the assumption that the floor is a continuous beam of four or five spans, free at each end. The position of wheel load for producing maximum moment, either positive or negative, will be found by trial -- that is, by placing a load P at various positions, (small load $\frac{1}{4}$P 14' -0" to the right), finding the resulting moments over the supports by moment distribution, or "Hardy Cross" and the moment under the load P by statics. From these data a table may be constructed which will show the approximate position of the load for producing maximum moment, either positive or negative. The calculations for the various positions are shown on the sheets following. Since the extreme end of each beam arrangement is considered free, some labor in distribution may be saved by using a modified distribution factor at the first and last intermediate girders.

The stiffness factor "F" for each floor span, considered as a beam, will be $\frac{EI}{L}$. Where E is the same for all members, the stiffness factors for the several members will be proportional to the quotient $\frac{I}{L}$ for any member. The distribution factor for a given member at any joint will be equal to the stiffness factor of the member divided by the sum of the stiffness factors of all the members intersecting at that joint or

$$\text{Distribution Factor} = \frac{F}{\Sigma F}.$$

For the floor, the length, moment of inertia, and modulus of elasticity for each span are the same; therefore, the stiffness factors will be equal where the initial distribution starting conditions are fixed at each end of each span. Where one end is fixed and the other free, as at the first and last supports, the corresponding beam is only three-fourths as rigid, and the modified distribution factors are:

$$.75 \div 1.75 = .428 \text{ and } 1.00 \div 1.75 = .572.$$

For the joints in between, the distribution factors are:

$$1 \div 2 = .500$$

For a beam loaded with a single concentrated load placed a distance "a" from the left end and a distance "b" from the right end, the fixed end moments are:

$$MF_a = Pa\frac{b^2}{L^2} \text{ and } MF_b = Pb\frac{a^2}{L^2}.$$

If the left end is now freed, the moment at the right end will be increased by half the amount required for fixity at the left end. This gives the initial condition for moment distribution at the right joint.

The calculations for the fixed end moment at the first intermediate girder are given here as illustrations:

$$"a" = 3 \text{ ft}, \ "b" = 5 \text{ ft}, \ L = 8 \text{ ft}$$
$$MF_a = P \times (3/8)\ L \times (5/8)^2 = .1462PL$$
$$MF_b = P \times (5/8)\ L \times (3/8)^2 = .088\ PL.$$

With the "a" end free, the fixed end moment at the "b" end becomes:

$$.088\ PL + \tfrac{1}{2}(.1465)\ PL = .1613\ PL.$$

The sign convention followed here considers the joint as the free body. Moments which tend to rotate the joint right hand are designated as positive; those which tend to rotate the joint left hand are designated as negative.

After distribution has been carried through four cycles, the columns are summed, and these sums taken as being the correct moment on the material of the joint. The end reaction and the bending moment under the load are computed by the principles of statics. Using the first distribution as an example again, we have,

$$\Sigma M_b = 0 = R \times L - P \times (5/8)L + .0851\ PL$$
$$\therefore R = .540\ P.$$

The positive bending moment under the load "P" is:

$$P \times (3/8)L = .540\ P \times .375L = .2025\ PL.$$

This process was repeated until enough data had been obtained to show definitely the greatest values which the bending moment over the support and the bending moment under the load would reach.

TABLE I

Unit Load, at Dist. From Left End	Moment Under Wheel	Moment Over First Intermediate Girder
3 ft	.2025PL	-.0851PL
3½	.2055PL	-.0919PL
4	.2027PL	-.0946PL
4½	.1972PL	-.0993PL
5	.1730PL	-.0982PL

From these values it may be readily seen that the positive moment will not exceed .21PL, and the negative will not exceed .100PL.

Since flexibility at the first support increases the positive moment under the wheel, the assumptions previously made will be followed and the positive moment will be taken as .21PL. For the negative moment over the first girder we consider that in reality some restraint will be provided by the supporting columns which in turn will increase the negative moment at this section. As the greatest fixed end moment here may be seen from the computation sheets to be about .19PL, the probable maximum will lie between .100PL and .20PL. We shall take the negative moment coefficient as .150PL.

For the uniformly distributed dead load, the following moment coefficients will be used: over supports, M = 1/8 WL; near the center, M = 1/10 WL.

The effective widths in moment and shear over which the wheel load is assumed distributed are computed from the A. R. E. A. formulas:[3]

$$E_M = (2/3)S + T \quad \text{but not over 6 ft.}$$

$$E_M = 2/3 \times 6.75 + 1.33 = 5.83 \text{ ft.}$$

Assuming a total floor depth of 12 inches and an effective depth of 10 inches, we have

$$E_s = 3.33 \times d + 1.33 \quad \text{but not over 6 ft.}$$

[3] H. Sutherland and W. W. Clifford, Reinforced Concrete Design (New York: John Wiley & Sons, Inc., 1926), pp. 145-147.

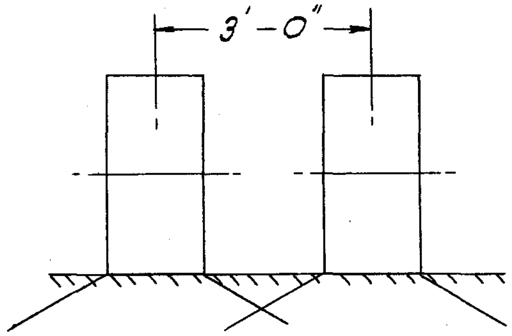
$$E_s = 3.33 \times 10/12 + 1.33 = 4.11 \text{ ft.}$$

In designing the floor slab, 30% will be added to the live load moments and shears to allow for impact.

These effective widths must be modified by the consideration that under certain circumstances the adjacent wheels of passing trucks might approach as close to one another as three feet, in which case the effective widths will overlap. To allow for this, we use the following:

$$E_M = \tfrac{1}{2}(5.83 + 3) = 4.42 \text{ ft}$$

$$E_s = \tfrac{1}{2}(4.11 + 3) = 3.55 \text{ ft.}$$



$PM = 16000/4.42 = 3610$ lb large wheel

$P_{1-M} = 4000/4.42 = 920$ lb small wheel

$P_v = 16000/3.55 = 4507$ lb large wheel

$P_{1-v} = 4000/3.55 = 1126$ lb small wheel.

Taking reinforced concrete as weighing 150 lb per cu ft and the surface paving as weighing 20 lb per sq ft, we have the following values for dead and live load moments:

U. D. L.-M	$-1/8\ WL = 1/8 \times 170 \times 8 \times 8$	=	-1360 lb-ft
-M	$1/10\ WL = 1/10 \times 170 \times 8 \times 8$	=	1090 lb-ft
C. L. L.-M	$-.150\ PL = -.150 \times 3610 \times 8$	=	-4330 lb-ft
-M	$.210\ PL = .210 \times 3610 \times 8$	=	6060 lb-ft

Greatest:

$$+M = 1090 + 1.3 \times 6060 = 8970 \text{ lb ft}$$

$$-M = -1360 + 1.3 \times 4330 = 6990 \text{ lb ft}$$

$$V = 574 + 1.3 \times 4507 = 6433 \text{ lb}$$

$$d = \sqrt{\frac{M}{K}} = \sqrt{\frac{8970}{172.3}} = 7.22 \text{ in}$$

$$d = \frac{V}{bjv} = \frac{6433}{12 \times .87 \times 75} = 8.22 \text{ in.}$$

The slab will be made 10 inches effective depth as assumed. The required area of steel over the supports will be:

$$A_s = \frac{M}{f_s jd} = \frac{6990 \times 12}{18000 \times .87 \times 10} = .535 \text{ in per ft.}$$

The required area of steel at the center will be:

$$A_s = \frac{M}{f_s jd} = \frac{8970 \times 12}{18000 \times .87 \times 10} = .689 \text{ in per ft.}$$

Bond area at support will be:

$$O = \frac{V}{ujd} = \frac{6433}{125 \times .87 \times 10} = 5.92 \text{ in per ft at support.}$$

For the first span the steel required is as follows:

Table II

	Support	P.I.	Center
Mom ft. lbs.	6990	----	8970
Shear lbs.	6474	4592	2253
A_s Top in.	.535	----	----
A_s Bot in.	----	----	.689
0 Top in.	5.92	4.21	----
00 Bot in.	----	----	2.49

In the preceding analysis, continuity of the slab was assumed in arriving at the moments over the supports. It will, therefore, be necessary to check for the moments existing in the adjacent spans to provide for the steel necessary to insure this continuity.

With a large wheel in the first span $4\frac{1}{2}$ feet from the left girder, the negative moment over the first intermediate support will be very nearly a maximum. For this condition, the adjacent span has the following end moments and shears:

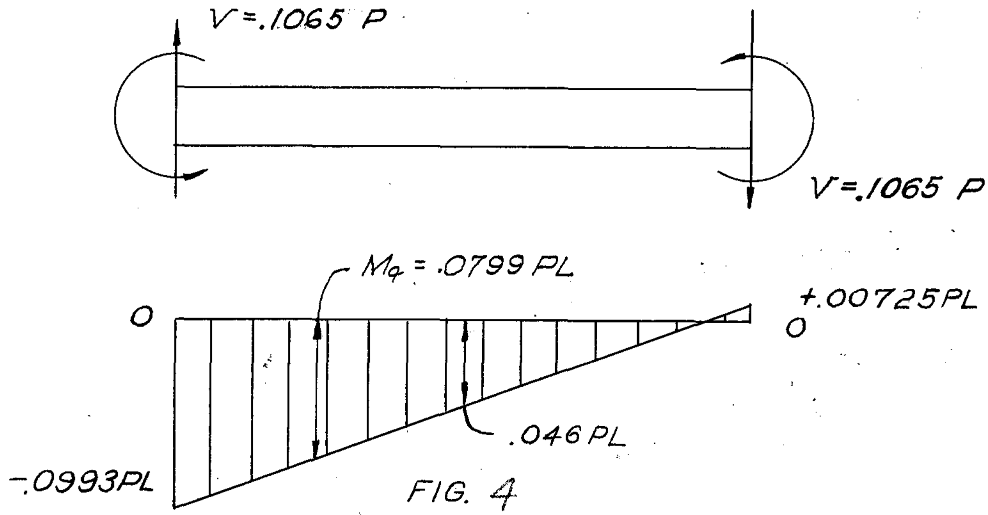


FIG. 4

At the quarter point of the beam the dead load bending moment will be quite small. At this point the negative moment due to continuity will be

$$M = -.0799 \times 3610 \times 8 = -2305 \text{ ft lb.}$$

The required area of steel for tension will be

$$A_s = \frac{2305 \times 12}{18000 \times .87 \times 10} = .177 \text{ sq in.}$$

To provide for this a ½-inch-round bar placed every 10 inches of floor width will be carried through the top of the floor slab for the entire length of the bridge. This provides a slight excess of tension area but helps to reduce the bond requirements at the supports.

Considering now the moments due to loads placed on the span in question, it may be seen that at the center, tension area governs, at the supports, bond governs. As v was used $.03f'_c$ in computing the depth of the floor slab, special anchorage will be required, and 7/12 of the A_s used at the center must be carried through to the supports.

For the intermediate spans the following bars will be used: on the bottom, 7/8-inch-round bars 10 inches o.c. running clear through from support to support; on the top, ½-inch-round bars 10 inches o.c. running clear through from support to support; also on top, 7/8-inch-round bars 8 inches o.c. over the supports from P.I. to P.I. and hooked.

For the end spans, the same through bars will be used at the top and bottom; at the extreme end, $\frac{1}{2}$-inch-round bars 10 inches o.c. will be used from the support to the quarter point at the bottom to provide for positive bending moment bond area.

P — 3' — 5' — $\frac{1}{4}P$

R = .54P

Mp = .2025PL

.428	.572	.50	.50	.50	.50	.572	.428
-.1613PL	.0000	.0000	+.0176PL	-.00342PL	.0000	.0000	.0000
+.0690	+.0923	-.0088	-.0088	+.00171	+.00171	.0000	.0000
.0000	-.0044	+.0461	+.00086	-.00440	.0000	+.00086	.0000
+.0019	+.0025	-.0235	-.02350	+.00220	+.0022	-.00049	-.00037
.0000	-.0118	+.0012	+.00110	-.0118	-.00049	+.00110	.0000
+.00505	+.00675	-.00115	-.00115	+.00615	+.00615	-.00063	-.00047
.0000	-.00057						
+.00024	+.00033						
-.0851PL	+.0851						

P — $3\frac{1}{2}$' — $4\frac{1}{2}$ — $\frac{1}{4}P$

R = .4711P

Mp = .2055PL

.428	.572	.50	.50	.50	.50	.572	.428
-.1770PL	.0000	.0000	+.0310PL	-.0072PL	.0000	.0000	.0000
+.0758	+.1012	-.0155	-.0155	+.0036	+.0036	.0000	.0000
.0000	-.0078	+.0506	+.0018	-.0078	.0000	+.0018	.0000
+.00334	+.00446	-.0262	-.0262	+.0039	+.0039	-.0010	-.0008
.0000	-.01310	+.00223	+.0020	-.0131	-.0005	+.0020	.0000
+.0056	+.00750	-.00211	-.00211	+.0068	+.0068	-.0011	-.0009
.0000	-.0010						
+.0004	+.0006						
-.0919	+.0919						

P $\frac{1}{4}P$

4' 4'

R = .4054P Mp = .2027PL

.428	.572	.50	.50	.50	.50	.572	.428
-.188PL	.0000	.0000	+.0351PL	-.0117PL	.0000	.0000	.0000
+.0810	+.1070	-.0175	-.0175	+.00585	+.00585	.0000	.0000
+.0000	-.0088	+.0535	+.00292	-.0088	.0000	+.0029	.0000
+.0038	+.0050	-.0282	-.0282	+.0044	+.0044	-.0017	-.0012
.0000	-.0141	+.0025	+.0022	-.0141	-.0008	+.0022	.0000
+.0081	+.0060	-.00235	-.00235	+.00745	+.00745	-.0013	-.0009
.0000	-.0012	+.00300	+.00745	-.00120	-.00065	+.0037	.0000
+.0005	+.0007	-.00522	-.00522	+.00092	+.00092	-.0021	-.0016
-.0946	+.0946						

P $\frac{1}{4}P$

4½' 3½'

R = .3507P Mp = .1972PL

.428	.572	.50	.50	.50	.50	.572	.428
-.1921PL	.0000	.0000	+.0369PL	-.0168PL	.0000	.0000	.0000
+.0821	+.1100	-.0184	-.0185	+.0084	+.0084	.0000	.0000
.0000	-.0092	+.0550	+.0042	-.0092	.0000	+.0042	.0000
+.0039	+.0053	-.0296	-.0296	+.0046	+.0046	-.0024	-.0018
.0000	-.0148	+.0026	+.0023	-.0148	-.0012	+.0023	.0000
+.0063	+.0085	-.00245	-.00245	+.0080	+.0080	-.0013	-.0010
.0000	-.0012	+.0042	+.0040	-.0012	-.0006	+.0040	.0000
+.0005	+.0007	-.0041	-.0041	+.0009	+.0009	-.0023	-.0017
-.0993	+.0993	+.00725	-.00725				

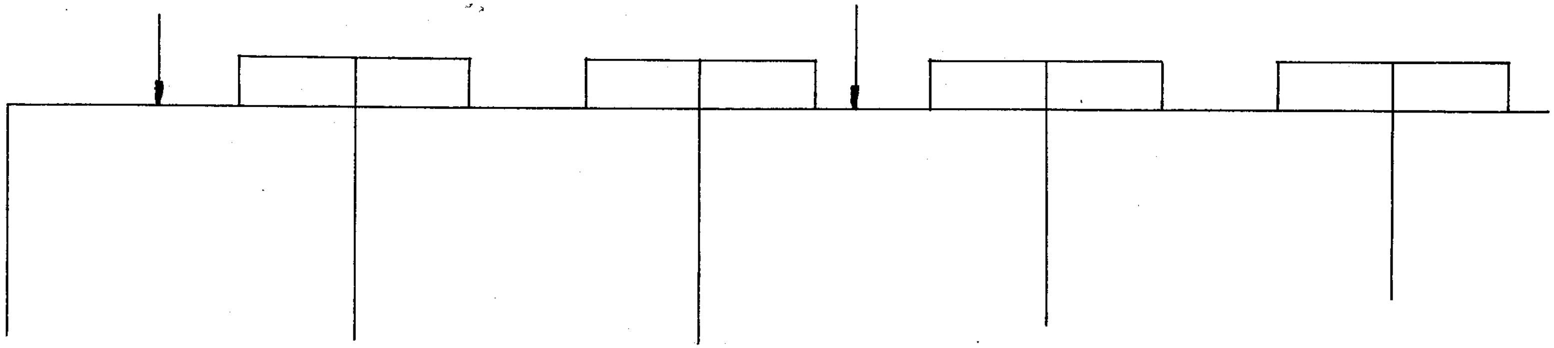
P   ½P

5'   3'

R = .2768P

Mp = .173PL

428	.572	.50	.50	.50	.50	.572	.428
-.1906PL	.0000	.0000	+.0366PL	-.0220PL	.0000	.0000	.0000
+.0816	+.1090	-.0183	-.0183	+.0110	+.0110	.0000	.0000
.0000	-.0092	+.0545	+.0055	-.0092	.0000	+.0055	.0000
+.0039	+.0053	-.0300	-.0300	+.0046	+.0046	-.0031	-.0024
.0000	-.0150	+.0027	+.0023	-.0150	-.0016	+.0023	.0000
+.0064	+.0086	-.0025	-.0025	+.0083	+.0083	-.0013	-.0010
.0000	-.0012	+.0043	+.0041	-.0012	-.0006	+.0041	.0000
+.0005	+.0007	-.0042	-.0042	+.0009	+.0009	-.0023	-.0018
-.0982	+.0982	+.0065	-.0065				



STEEL FOR RAILING

Railing is designed to withstand a vertical load of 100 pounds per foot and a horizontal load of 150 pounds per foot.

For the top railing the vertical load will be carried by the balusters. The horizontal load will be carried by the top rail in bending. The bending moment will be a maximum at the center. Here The Bending Moment is

$$M = 150 \times \frac{8x8}{8} = 1200 \text{ lb ft}$$

$$A_s = \frac{1200x12}{18000x.86x8} = .116 \text{ sq in.}$$

We will use a one half inch round bar in each corner.

The greatest Bending Moment on the post will occur where it joins the curb. The load on the post will equal two reactions, one from each top rail joining it. The Maximum Bending Moment will be:

$$M = 1200 \times 2.4 = 2880 \text{ lb ft}$$

$$A_s = \frac{2880x12}{18000x.86x10} = .223 \text{ sq in.}$$

We will use a 5/8 inch round bar in each corner.

CHAPTER III

DEAD LOADS ON GIRDER

The precise calculation of the dead load moments and reactions for the girder is not practical by any of the usual methods. It is probable, however, that the values will lie somewhere between the two extremes obtained by the following assumptions. First, the assumption that the three supports for the girder all have the same elevation will result in a heavier load on each of the outside columns and a lighter load on the center column. Second, the assumption that the dead load on the arch rib is uniformly distributed over a strip parallel to the girder will require that the outer column should deflect more than the center one; that is, some vertical sway in the frame must be allowed for. The calculations for both assumptions will now be presented.

When a beam load is applied to the girder, some rotation at the base of the columns supporting it will take place. The exact amount of this cannot be determined readily by ordinary methods but will be estimated here on the assumption that a portion of the arch rib 6 feet long acts with the columns and girders to form a frame to which the method of moment distribution will be applied.

The stiffness factors for the various parts of the frame will be computed as follows:

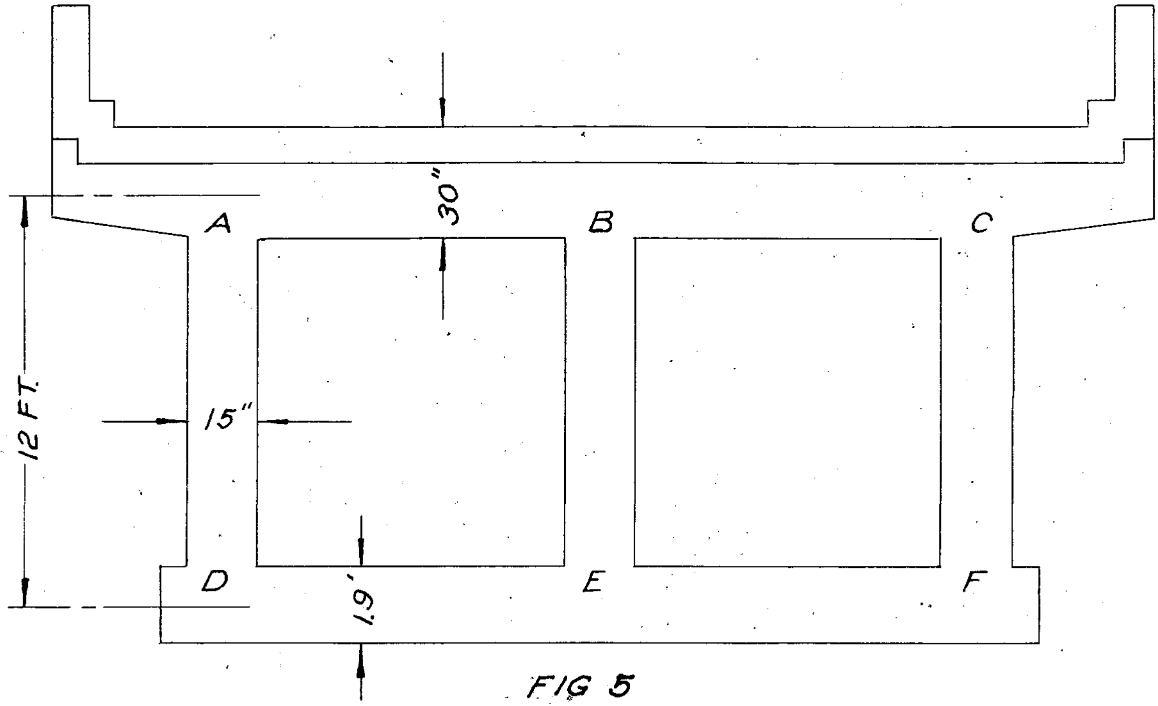


FIG 5

$$I_{girder} = \frac{(1.25) \times (2.5)^3}{12} = 1.63 + 10\% \text{ for floor} = 1.79 \text{ ft}^4$$

$$I_{column} = \frac{1.25 \times (1.25)^3}{12} = .203 \text{ ft}^4$$

$$I_{arch} = \frac{6 \times (1.9)^3}{12} = 3.43; \quad \frac{3.43}{.766} = 4.47 \text{ ft.}^4$$

$$F_{\text{for girder}} = 1.79 \div 10 = .179$$

$$F_{\text{for column}} = .203 \div 12 = .017$$

$$F_{\text{for arch}} = 4.47 \div 10 = .447$$

Distribution factors for long column bent

For joints A & C $\frac{F}{\Sigma F} = \frac{.179}{.196} = .914$ Use .91 girder

$\frac{F}{\Sigma F} = \frac{.017}{.196} = .086$ Use .09 column

For joint B $\frac{F}{\Sigma F} = \frac{.179}{.375} = .477$ Use .475 girders

$\frac{F}{\Sigma F} = \frac{.017}{.375} = .0453$ Use .05 column

For joints D & G $\frac{F}{\Sigma F} = \frac{.017}{.464} = .0366$ Use .04 column

$\frac{F}{\Sigma F} = \frac{.447}{.464} = .965$ Use .96 arch

For joint E $\frac{F}{\Sigma F} = \frac{.017}{.911} = .0187$ Use .02 column

$\frac{F}{\Sigma F} = \frac{.447}{.911} = .491$ Use .49 arch

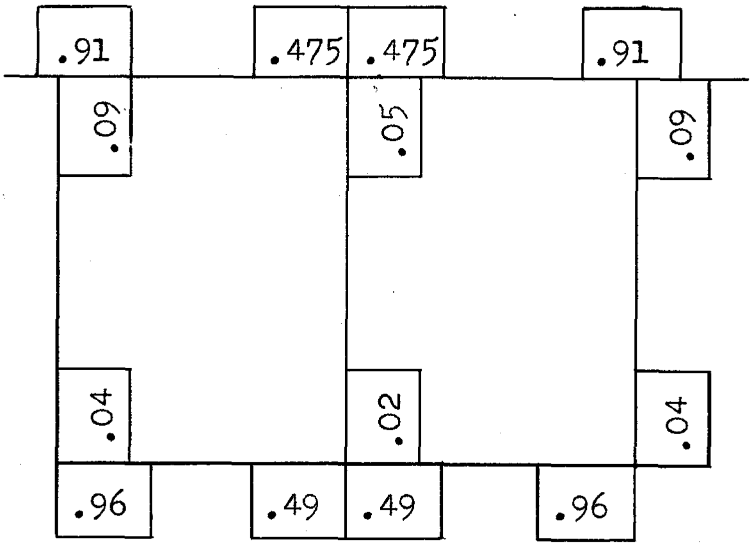


TABLE III

CALCULATION OF DEAD LOAD ON GIRDERS

Dimensions in feet, concrete 150 pounds per cubic foot.

Cantilever Dead Load

Post $2\frac{3}{4} \times 1 \times 1 \times 150$	=	412 lb	
Top Rail $\frac{10}{12} \times \frac{1}{2} \times 7 \times 150$	=	438 lb	
Balusters $\frac{1}{2} \times \frac{1}{2} \times 2 \times 7 \times 150$	=	525 lb	
Bottom Curb $1\frac{3}{4} \times \frac{3}{4} \times 8 \times 150$	=	1575 lb	
Floor $1 \times 4\frac{3}{4} \times 6.75 \times 150$	=	4810 lb	
Girder Rect. $2\frac{1}{4} \times 1\frac{1}{4} \times 5\frac{1}{2} \times 150$	=	2320 lb	
Triangle $\frac{1}{4} \times 1\frac{1}{4} \times 4\frac{3}{4} \times 150 \times \frac{1}{2}$	=	111 lb	Approx.
Paving $3\frac{3}{4} \times 8 \times 20$	=	600 lb	
		10,791 lb	

TABLE IV

MOMENT AT CENTER LINE OF OUTER COLUMN

Part	Weight	Moment Arm	Moment Ft Lb
Post	412	5 ft	2060.
Top Rail	438	5	2190.
Balusters	525	5	2625.
Bottom Curb	1575	4.625	7284.
Floor	4810	2.375	11424.
Girder Rect.	2320	2.750	6380.
Triangle	111	2.23	248.
Paving	600	1.875	1125.
			33,336 lb ft

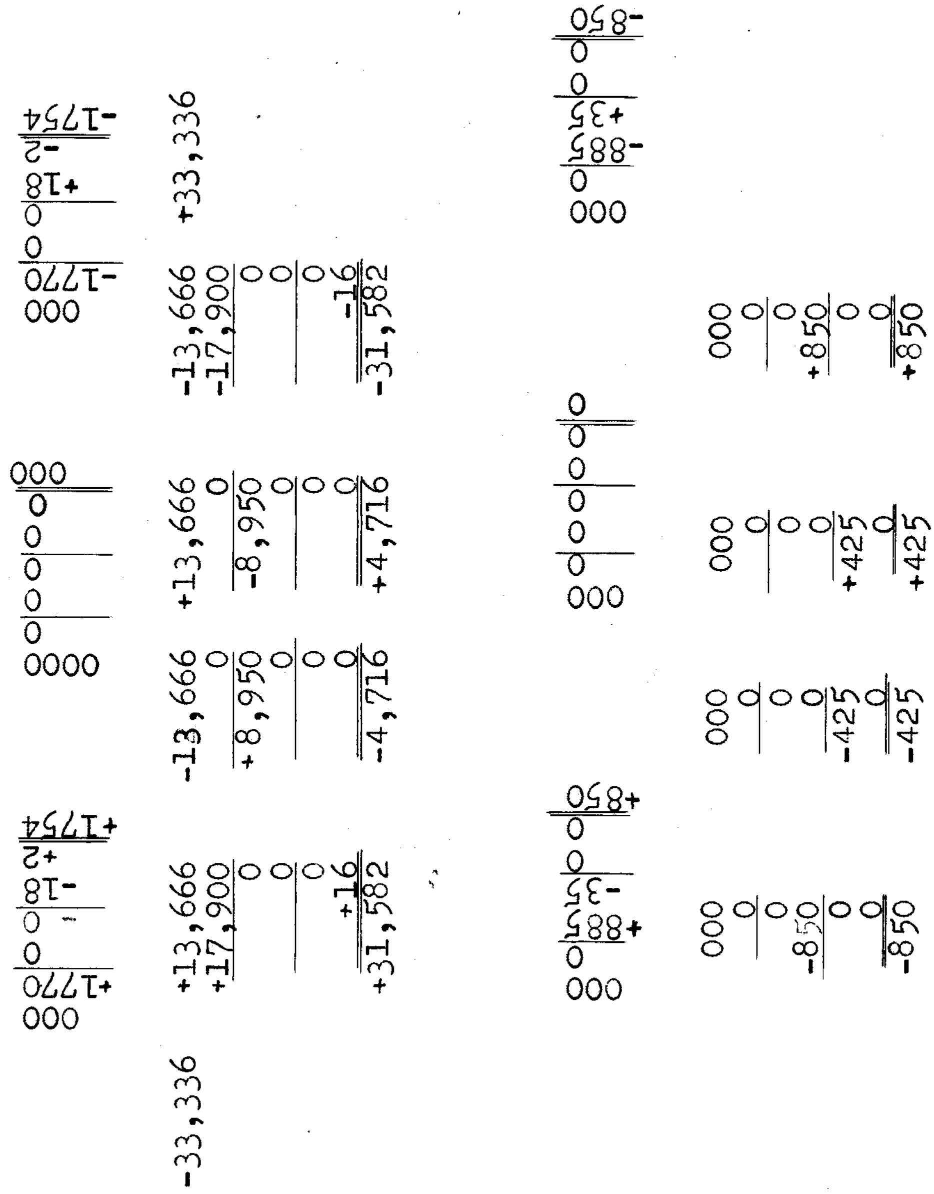
Dead Load on Girder, Weight per foot

Floor	$1 \times 6.75 \times 150$	=	1012 lb per foot
Paving	$1 \times 8 \times 20$	=	160
Girder	$1\frac{1}{4} \times 2\frac{1}{2} \times 150$	=	468
			1640 lb per foot

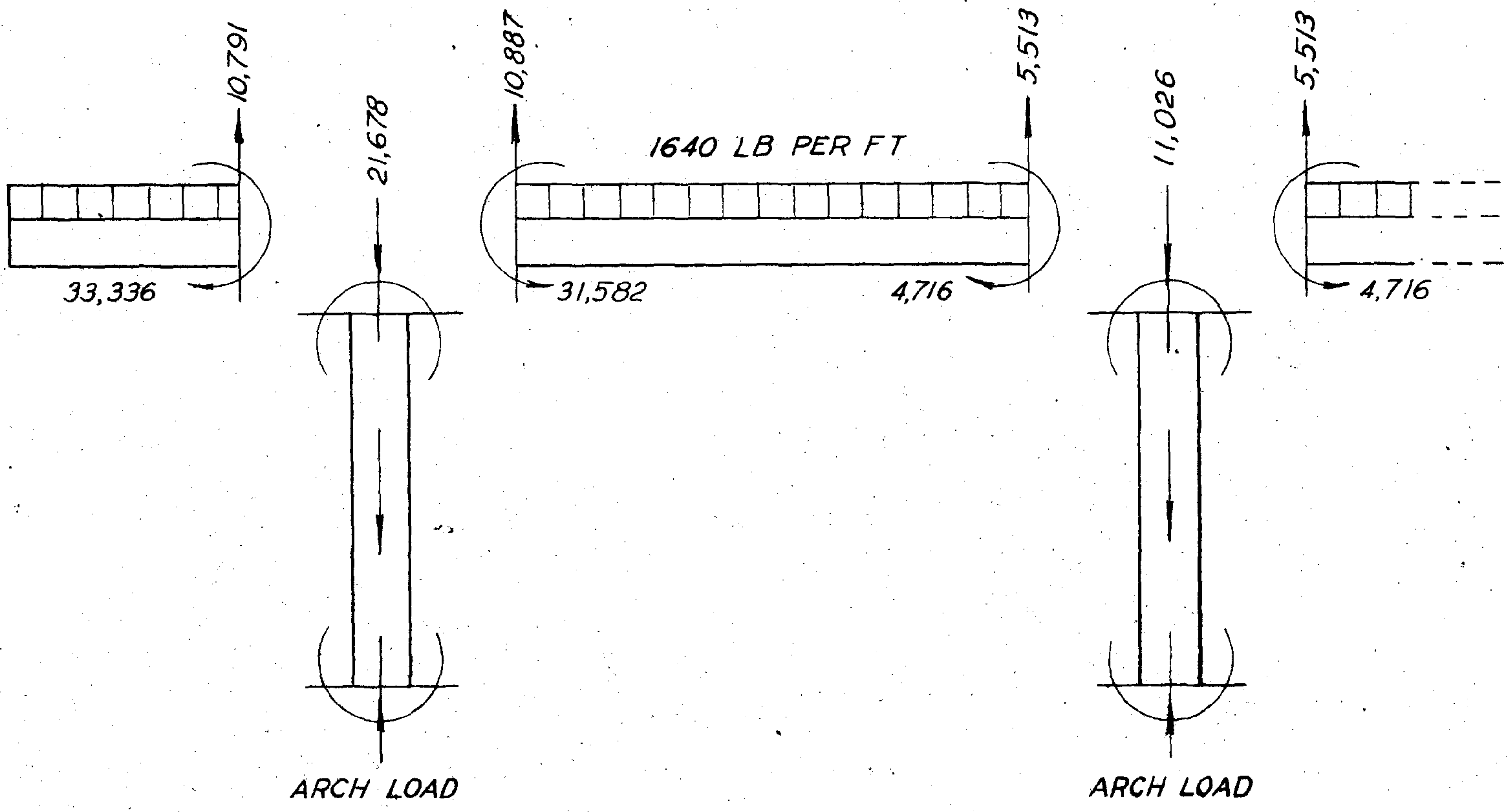
For the first dead load distribution assumption, we have the following fixed end moments:

Cantilever overhang	=	33,336 lb ft
Girder $\frac{Wl}{12}$ - $1640 \times 10 \times \frac{10}{12}$	-	13,666 lb ft

The moment distribution for this assumption is shown on the following sheet.



Dead Load Assumption No. 1



Assumption No. 1 – No Yielding of Supports
FIG 6

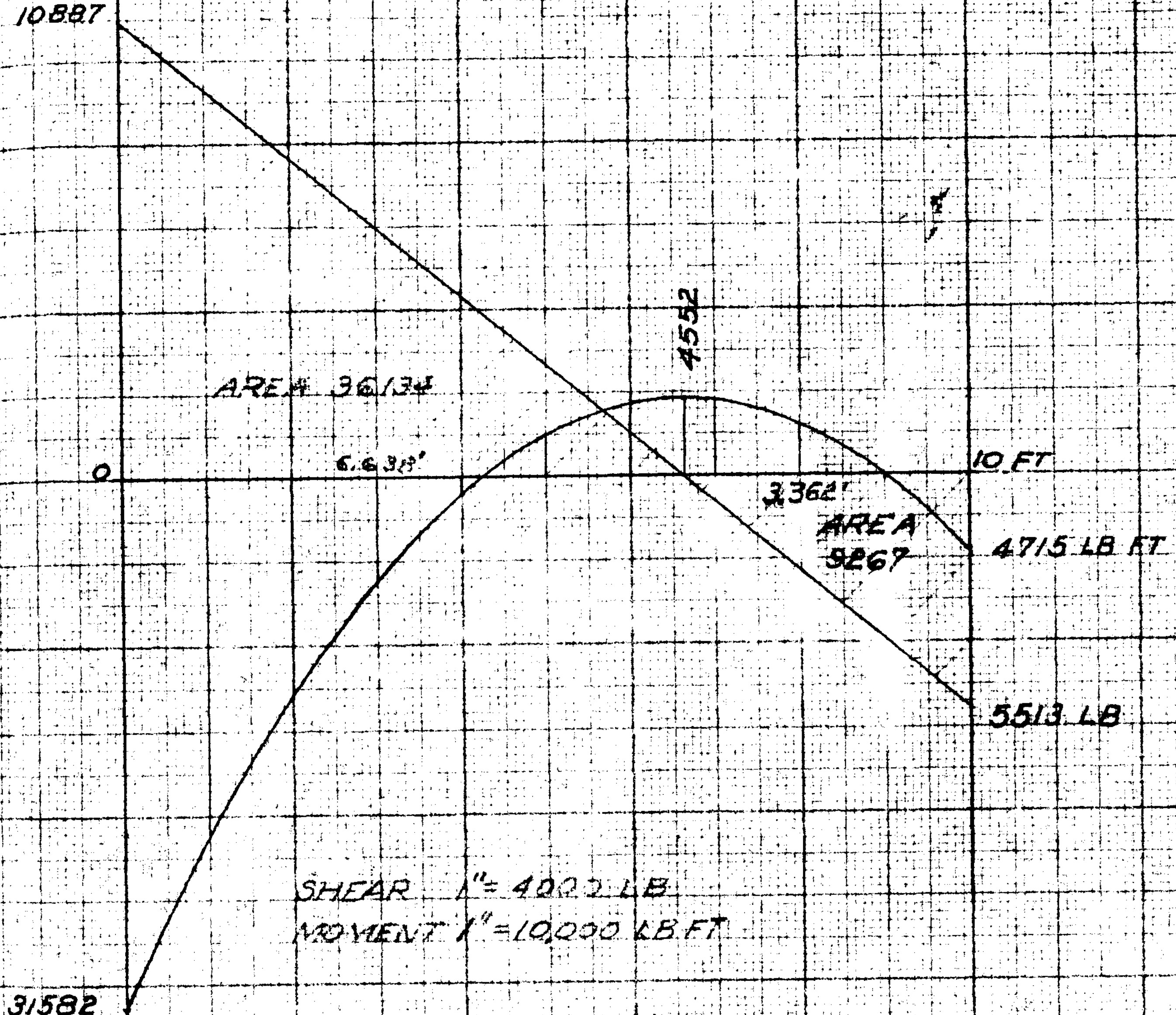


FIG 7

GIRDER
DEAD LOAD SHEAR AND MOMENT
ASSUMPTION NO 1

The second assumption as to the distribution of dead load on the arch is that the arch resistance is uniformly distributed over a strip parallel to the girder. To calculate the value of this, it will be necessary to know the weight of the columns and footings.

For the long column bent

$$\text{One column } 1\tfrac{1}{4} \times 1\tfrac{1}{4} \times 9\tfrac{1}{4} \times 150 = 2180 \text{ lb}$$

$$\text{One footing est. } 2 \times 2 \times 10/12 \times 150 = \frac{500}{2680} \text{ lb.}$$

This gives the total weight on the strip of arch

$$2 \times 10{,}791 + 3 \times 2680 + 2 \times 16400 = 62422 \text{ lb.}$$

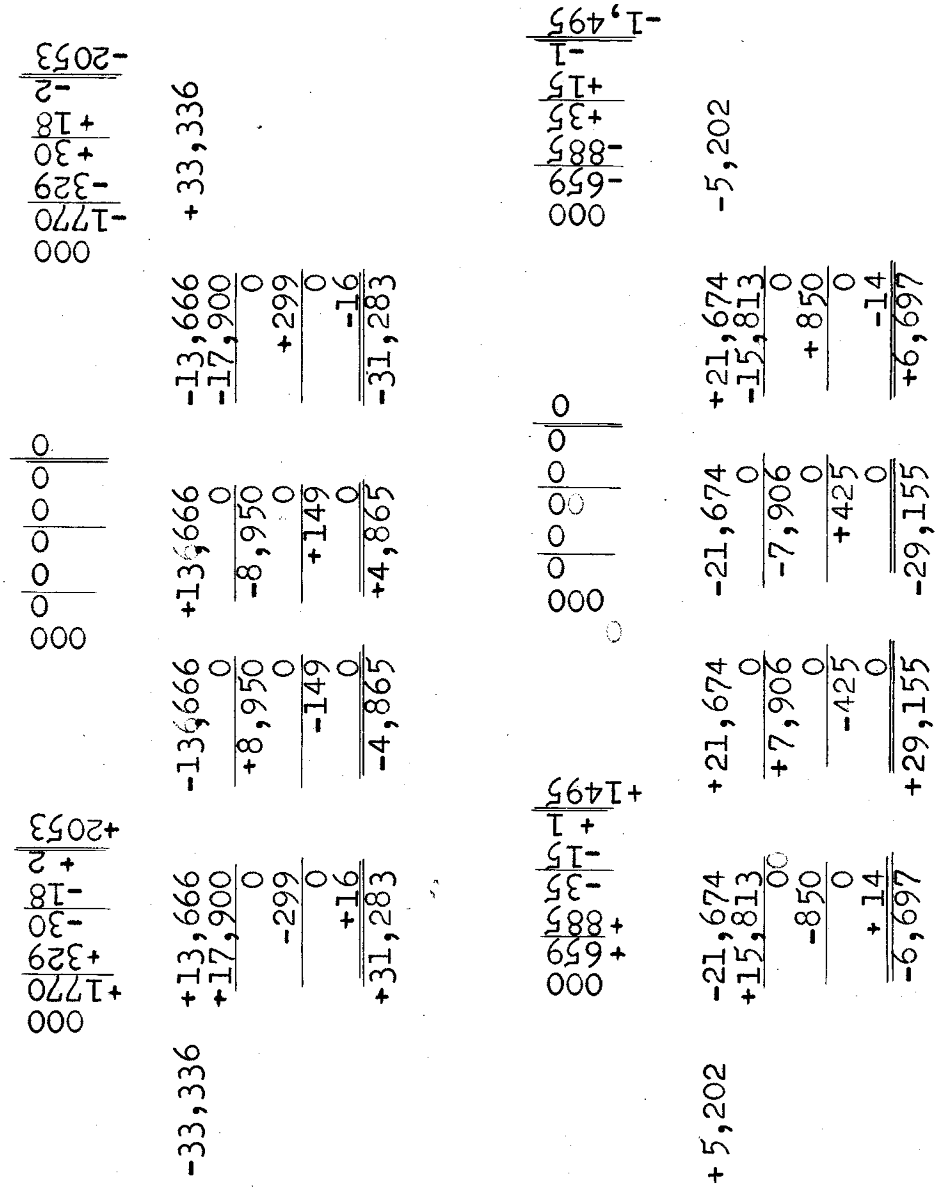
This distributed over 24 feet gives 2600.9 lb per foot.

This will give fixed end moments on the arch as a beam equal to

$$2600.9 \times 10 \times 10/12 = 21{,}674 \text{ lb ft.}$$

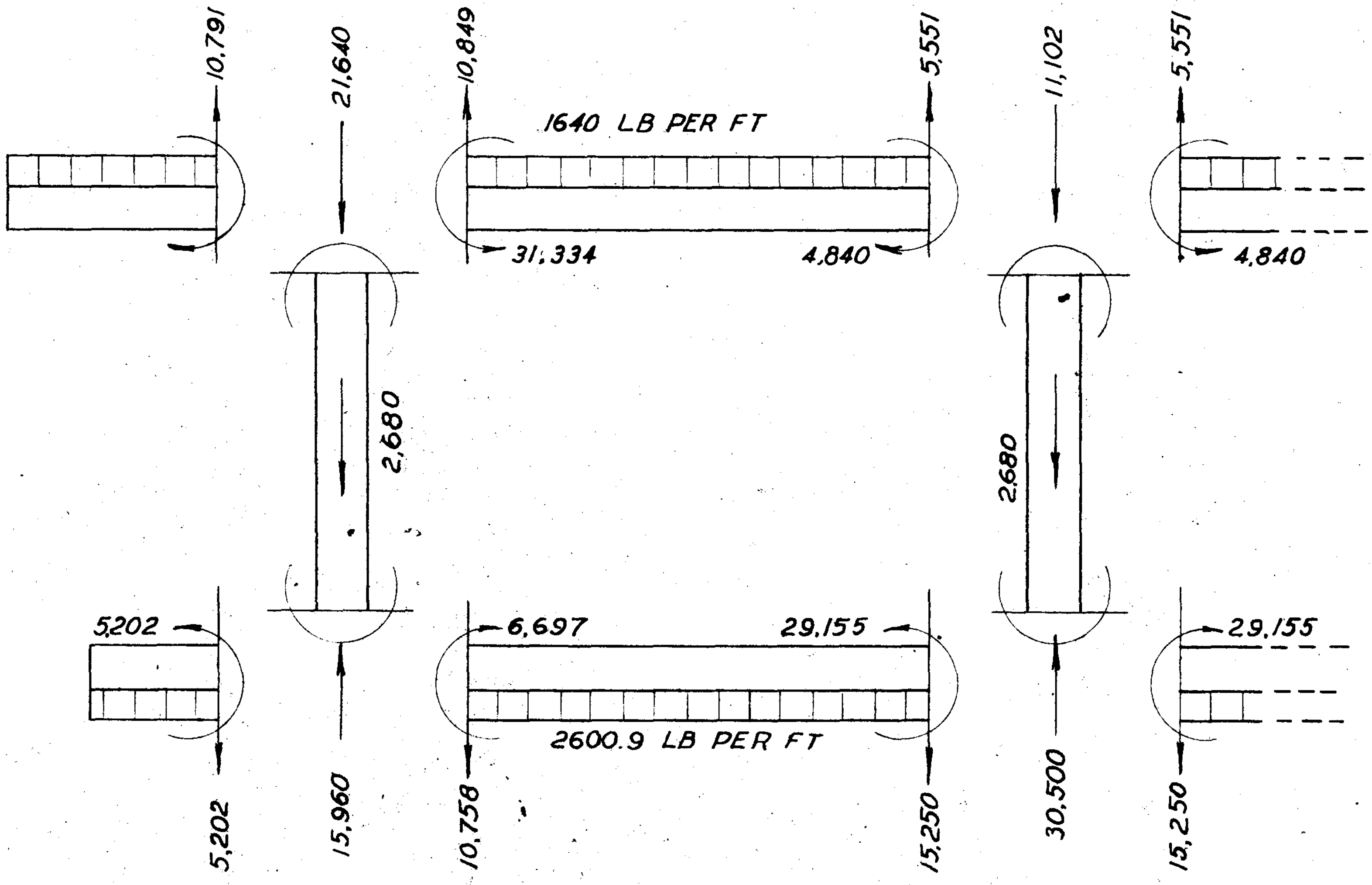
For the cantilever over-hang on arch rib we have

$$2601 \times 2 \times 1 = 5202 \text{ lb ft.}$$



Dead Load Assumption No. 2

No Vertical Sway



Assumption No. 2
No Sway Allowed
FIG 8

The shears obtained by assumption No. 2 with no vertical sway allowed show an unbalanced force on each column, an up-force on the center column, a down-force on each outer column. Enough vertical sway must take place to equalize the unbalance forces on each column and put it in equilibrium.

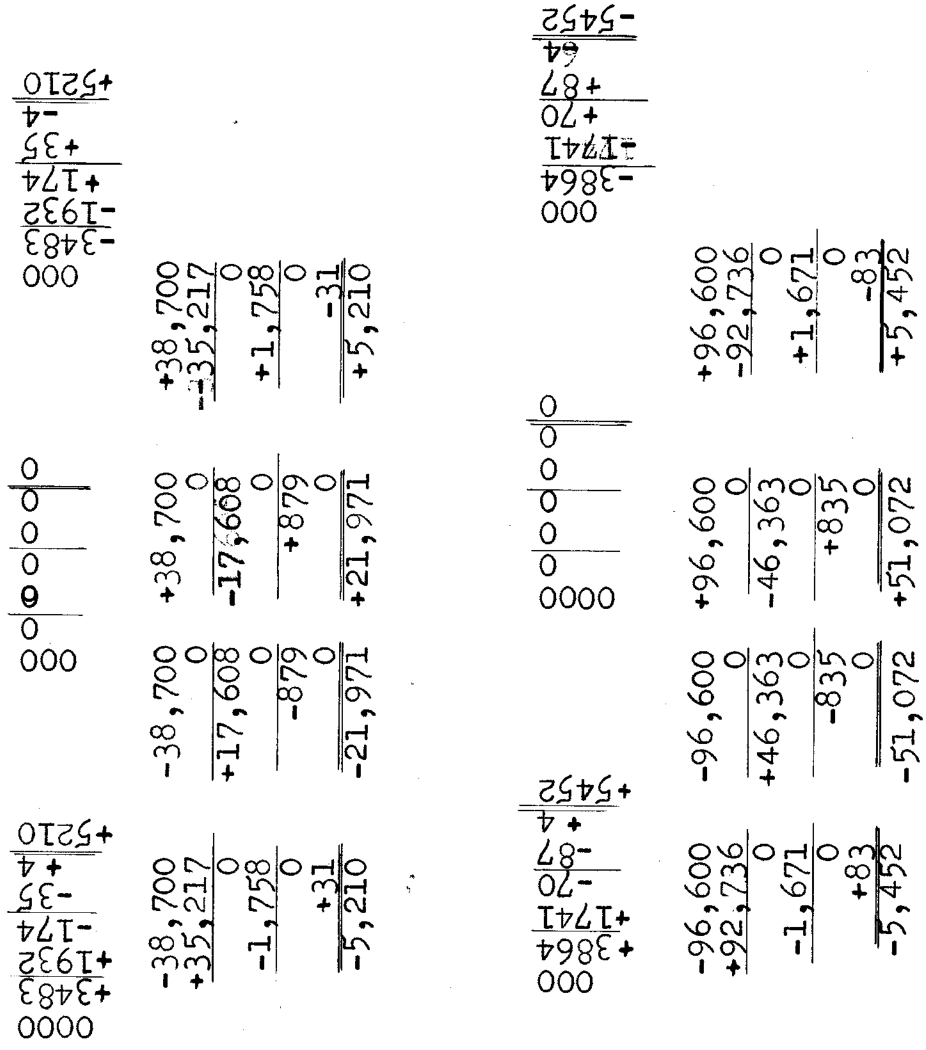
To obtain the amount of this sway, we assume an arbitrary amount, the end columns down, the center column up, no rotation of the girder ends. This gives certain fixed end moments which are distributed. The resulting shears on the girder are then computed. This gives a ratio between the moments and the shears in the frame for sway of the desired kind but of amount not known. From the unbalanced forces on the columns, obtained from assumption No. 2, the necessary sway shears may be seen, and from these, the correct bending moments introduced at the joints by permitting the necessary vertical sway to produce equilibrium.

Assuming a vertical sway deflection of .01 inches, we have the following fixed end moments for no rotation of girder ends:

$$MF = \frac{6\ EI}{L} = \frac{\Delta}{L}$$

$$\text{Girder } MF = \frac{6 \times 3{,}000{,}000 \times 144 \times 1.79}{10 \times 1200 \times 10} = 38{,}700 \text{ lb ft.}$$

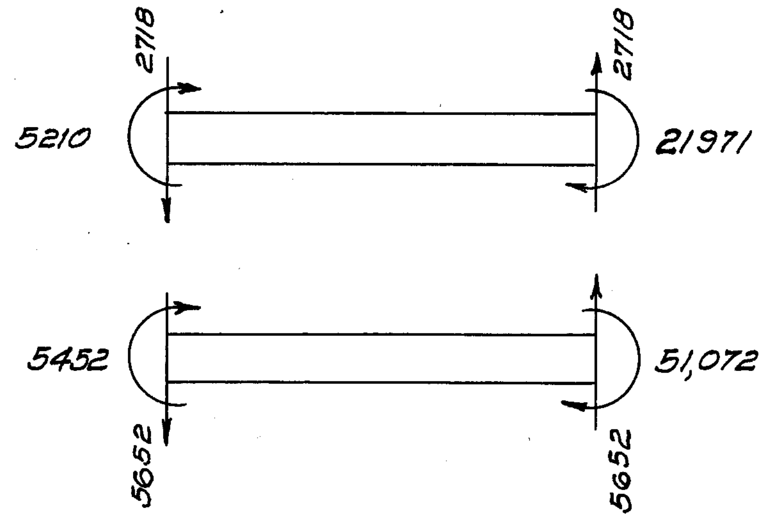
$$\text{Arch } MF = \frac{6 \times 3{,}000{,}000 \times 144 \times 4.47}{10 \times 1200 \times 10} = 96{,}600 \text{ lb ft.}$$



Assumed Vertical Sway

From the free body diagrams when no sway is permitted the unbalanced force on each outer column is 8,360 lb and on the center column 16,718 lb.

The assumed sway gives shears at the outer column of 5210 lb for the girders and 5652 lb for the arch.



Shears To Accompany Assumed Sway

Since there is required a shear correction at the outer column of 8360 lb, the assumed sway is nearly correct. The proportion to use will be

$$\frac{8360}{8370} = .999.$$

The algebraic sum of the moments without sway and the moments due to sway correction will give the final moments for assumption No. 2. This is shown on the following page.

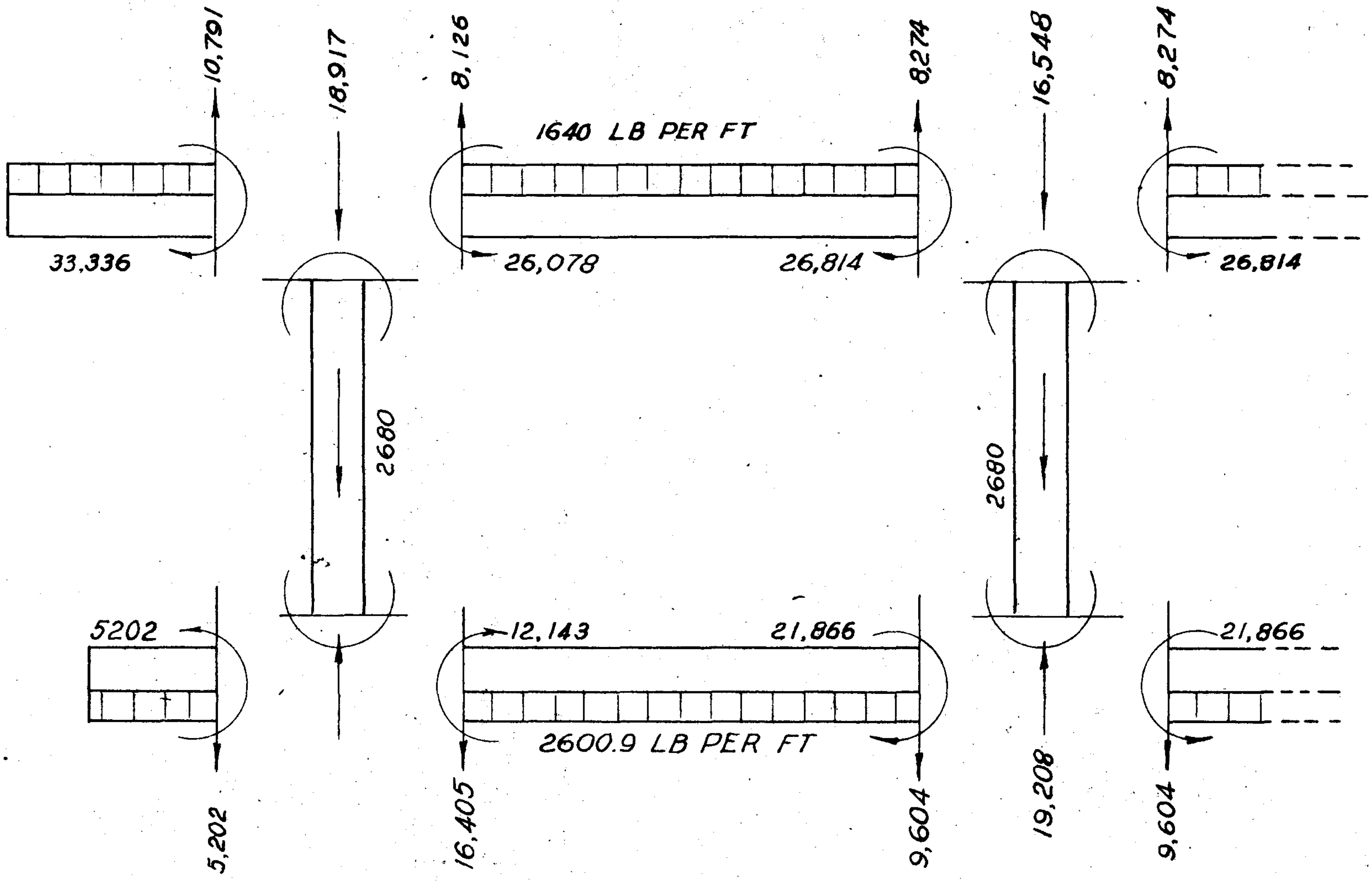
+31283	- 4865	+ 4865	-31283
- 5205	-21949	+21949	+ 5205
+26078	-26814	+26814	-26078

- 6697	+29155	-29155	+ 6697
- 5446	-51021	+51021	+ 5446
-12143	-21866	+21866	+12143

Moment Distribution Signs

Final Girder Moments

Assumption No. 2



Assumption No. 2
Final Shears and Moments
FIG 9

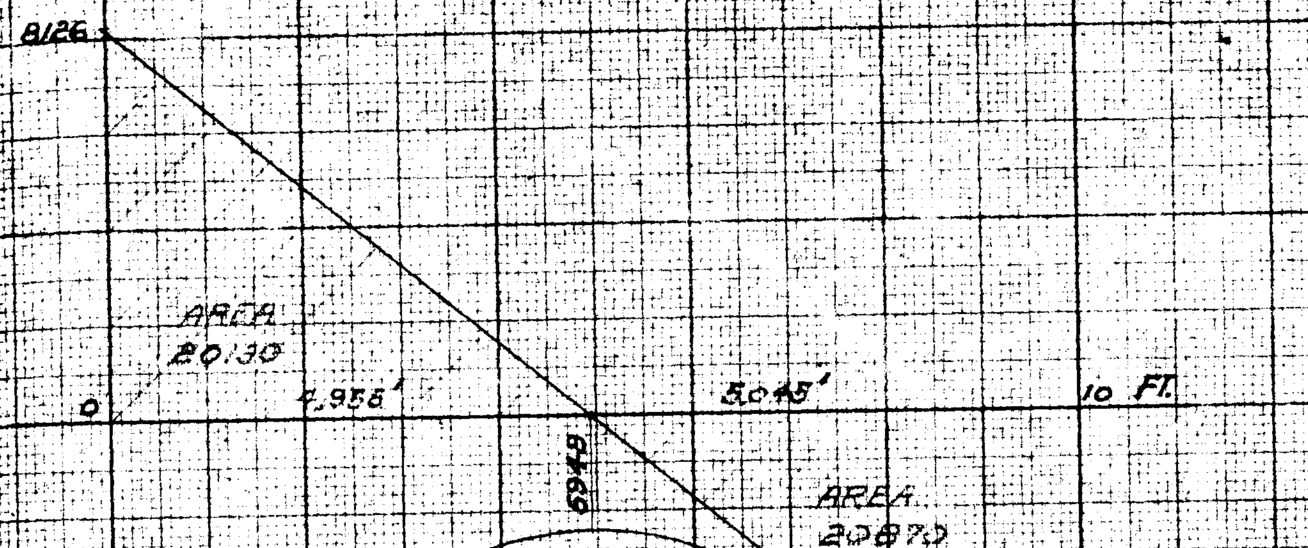


FIG 10

GIRDER
DEAD LOAD SHEAR AND MOMENT
ASSUMPTION NO 2

The girders over the center pier will have the same cross-sectional dimensions between columns as the other girders, but the cantilevers for these girders will be of thicker proportions for architectural reasons. The center column will be 18 inches square, while those on the outside will be 18 inches at the top and 20 inches at the pier. This means that the outside columns have variable moments of inertia. The stiffness and the carry-over factors for the outside column will be computed by the method given in Dr. Snow's "Concrete Notes."[4]

The columns will be considered as fixed at the pier top; hence, it will be necessary to compute the stiffness factors and the distribution factors only for the joints between the girder and the columns.

$$I_{girder} = \frac{1.25 \times \overline{2.5}^3}{12} = 1.628 + 10\% \text{ for floor} = 1.791 \text{ ft}^4$$

$$I_{center\ column} = \frac{1.5 \times \overline{1.5}^3}{12} = .422 \text{ ft}^4$$

$$F_{for\ girder} = 1.791 \div 10 = .1791$$

$$F_{center\ column} = .422 \div 18 = .0234$$

$$F_{outside\ column} = .447 \div 18 = .0249$$

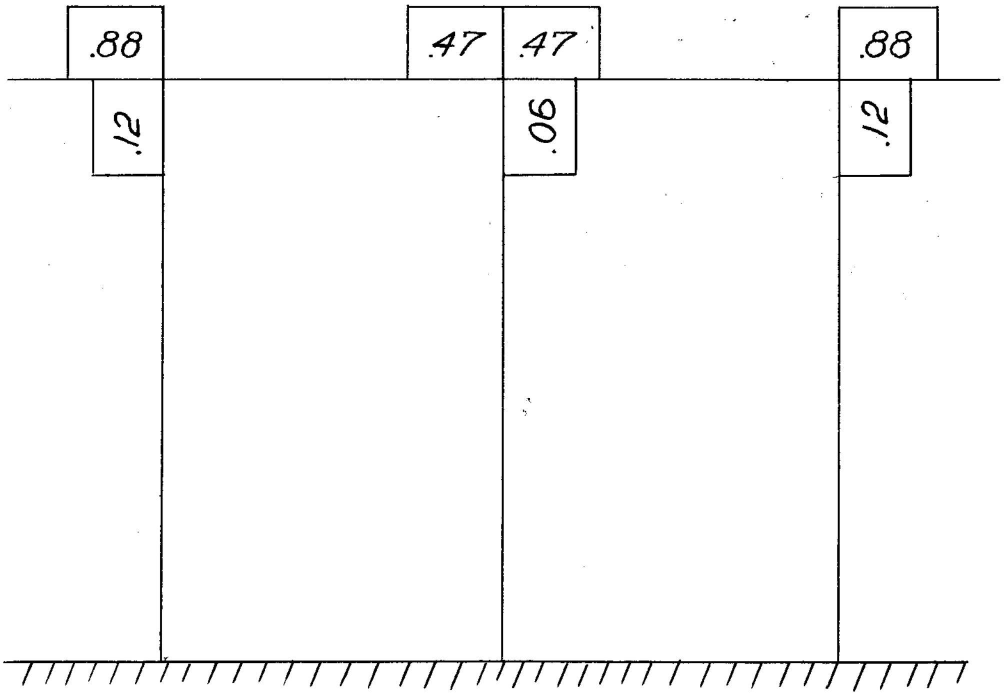
[4]F. C. Snow, Concrete Notes. Georgia School of Technology, class notes for courses in reinforced concrete, pp 108.

Joints A & C $\quad \frac{F}{\Sigma F} = \frac{.0249}{.204} = .122 \qquad$ Use .120 Column

$\frac{F}{\Sigma F} = \frac{.1791}{.204} = .878 \qquad$ Use .880 Girder

Joint B $\quad \frac{F}{\Sigma F} = \frac{.0234}{.3816} = .0614 \qquad$ Use .06 Column

$\frac{F}{\Sigma F} = \frac{.1791}{.3816} = .47 \qquad$ Use .47 Girder



Calculation of dead loads on pier girder

Cantilever

Lamp and stand (Estimated)	= 400 lb
Post Cap $1.25 \times \frac{1}{2} \times 3\frac{1}{2} \times 150$	= 328
Post $2\frac{3}{4} \times 1\frac{1}{2} \times 1 \times 150$	= 619
Top Rail $\frac{10}{12} \times \frac{1}{2} \times 3.5 \times 150$	= 219
Balusters $\frac{1}{2} \times \frac{1}{2} \times 2 \times 3\frac{1}{2} \times 150$	= 262
Bottom Curb $1\frac{3}{4} \times \frac{3}{4} \times 4.85 \times 150$	= 955
Floor $1 \times 4\frac{3}{4} \times 3.60 \times 150$	= 2560
Girder Rectangle $2\frac{1}{4} \times 1\frac{1}{2} \times 5\frac{1}{2} \times 150$	= 2780
Triangle $\frac{1}{4} \times 1\frac{1}{2} \times 4\frac{3}{4} \times 150 \times \frac{1}{2}$	= 134
Paving $3\frac{3}{4} \times 4.85 \times 20$	= 364
	8551 lb.

PIER - MOMENT AT CENTER LINE OF OUTER COLUMN

TABLE NO. V.

Part	Weight	Moment Arm	Moment Ft-Lb
Lamp	400	5 ft	2000
Post Cap	328	5	1640
Post	619	5	3095
Top Rail	219	5	1095
Balusters	262	5	1310
Bottom Curb	955	4.625	4417
Floor	2560	2.375	6080
G Rectangle	2780	2.750	7645
G Triangle	134	2.230	299
Paving	364	1.875	682
			28,263 ft-lb

Dead load on girder will be

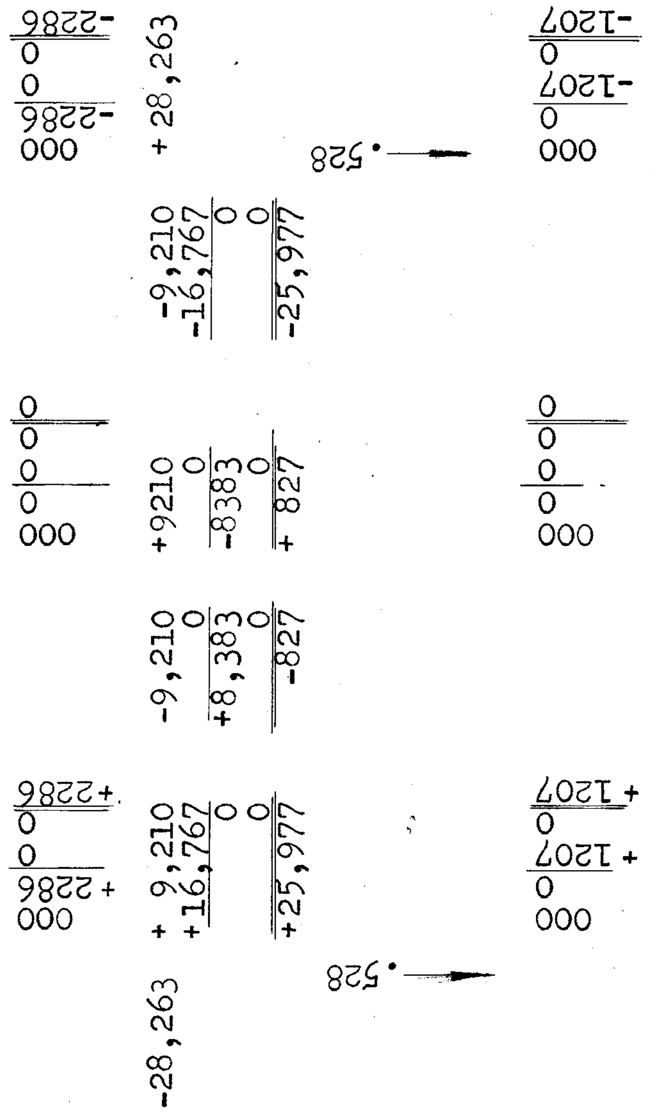
Floor	$1 \times 3.60 \times 150$ =	540
Paving	$1 \times 4.85 \times 150$ =	97
Girder	$1\frac{1}{4} \times 2\frac{1}{2} \times 150$ =	468
		1105 lb per ft.

For the dead load distribution, we have the following moments:

Cantilever = 23263 lb ft

$$\text{Girder } \frac{Wl}{12} = 1105 \times \frac{100}{12} = 9210 \text{ lb ft.}$$

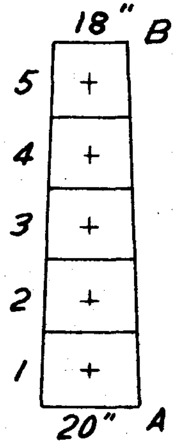
The distribution is shown on the following sheet:



Dead Load Distribution For Pier Girder

TABLE VI STIFFNESS FACTORS AND CARRY OVER FOR PIER GIRDER

1	2	3	4	5	6	7	8	9	10
POINT	x	b	I	M_L V_L	$\frac{M_L}{I}$	$\frac{M_L}{I}x$	M_R	$\frac{M_R}{I}$	$\frac{M_R x}{I}$
5	.9H	1.5166	.42654	.1	.23444	.210996H	.9	2.1100	1.899 H
4	.7H	1.5500	.43594	.3	.68816	.48171H	.7	1.6057	1.1240 H
3	.5H	1.5833	.44530	.5	1.1228	.56140 H	.5	1.1228	.56140H
2	.3H	1.6166	.45460	.7	1.5396	.46188H	.3	.6598	.19794H
1	.1H	1.650	.46406	.9	1.9394	.19394H	.1	.2155	.02155
					5.5244	1.9099H		5.7138	3.8039 H



$$a = \frac{\Sigma \text{col } 7}{\Sigma \text{col } 6} = \frac{1.9099H}{5.5244} = .34572H \qquad b = \frac{\Sigma \text{col } 0}{\Sigma \text{col } 9} = \frac{3.8039H}{5.7138} = .66573H$$

$$(\text{Carry Over})(A \text{ to } B) = \frac{\Sigma \text{col } 6 \times (a)}{\Sigma \text{col } 9 \times (b)} = \frac{1.9099H}{3.8038H} = .5021$$

$$(\text{Carry Over})(B \text{ to } A) = \frac{\Sigma \text{col } 9 \times (H-b)}{\Sigma \text{col } 6 \times (H-a)} = \frac{1.90995}{3.6145} = .528$$

Stiffness Factor $$F_B = \frac{1}{4dx[\Sigma \text{col } 9 - C(B-A)\,\Sigma \text{col } 6]} = \frac{1}{.8H[2.797]} = \frac{.447}{H}$$

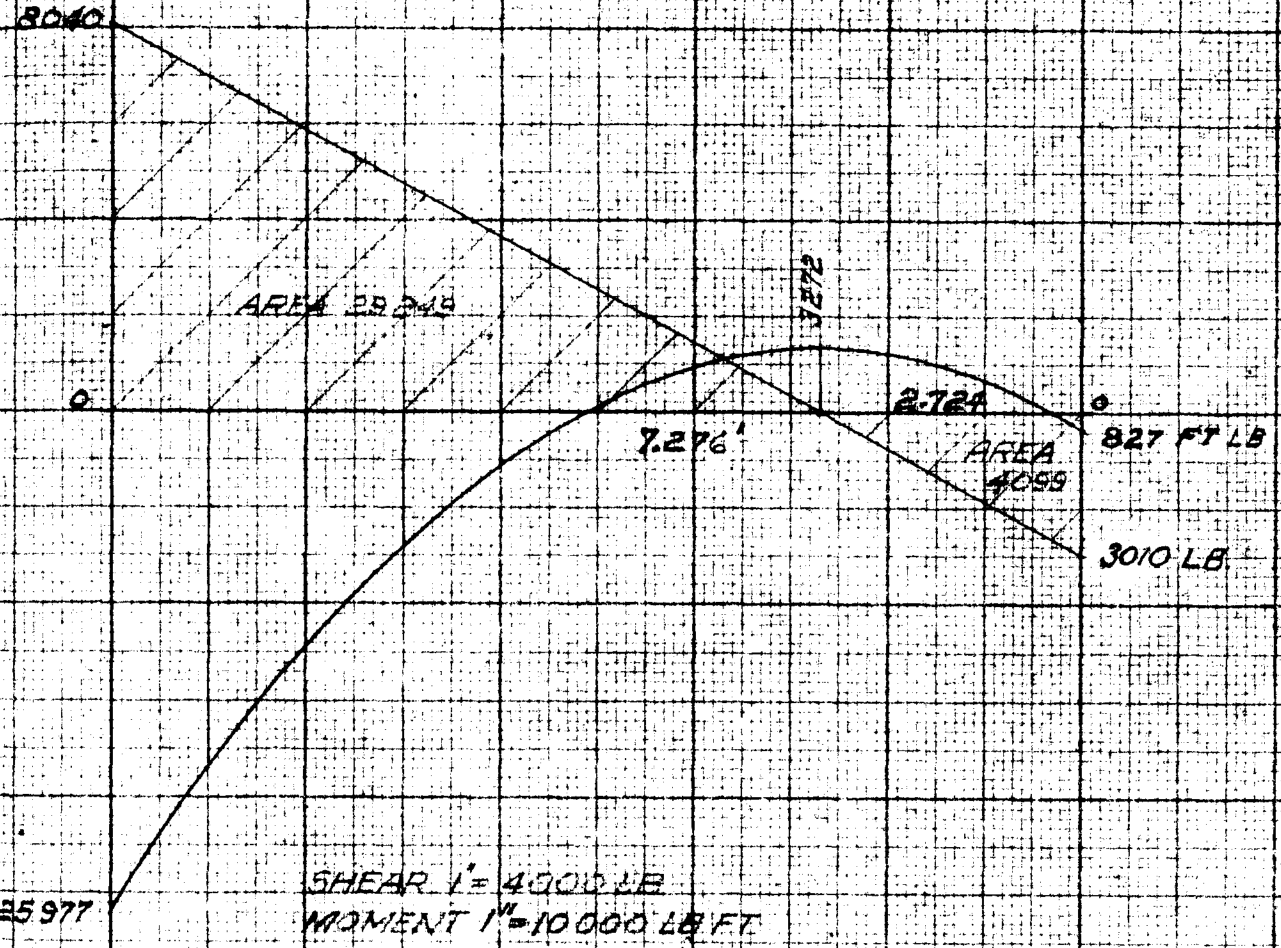


FIG 11

PIER GIRDER
DEAD LOAD SHEAR AND MOMENT

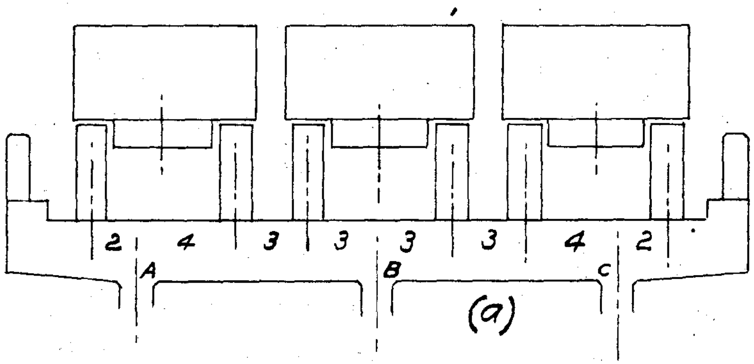
CHAPTER IV

LIVE LOADS ON GIRDER

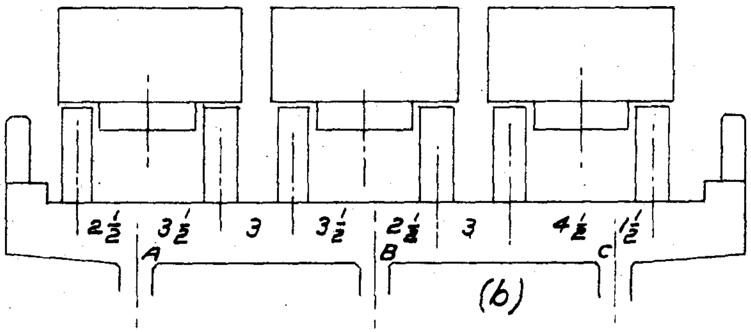
On the following sheets are shown the calculations for maximum bending moments and maximum shears at certain critical sections of the girder. By maximum is meant, in many cases, approximate maximum, as the exact maximum is frequently difficult to obtain and very little different numerically from the approximate maximums used for selecting the required sizes of reinforcing steel.

On Fig. 12,a through k, are shown the positions of the trucks taken as producing the various maximums stated. The solution is by moment distribution or "Hardy Cross". Each distribution is shown on a sheet by itself, and immediately following it is a sheet showing the calculations for the fixed-end moments, the correction for sidesway where used, and a bending moment diagram for the girder in question. The method of sidesway correction used is that presented by Dr. F. C. Snow in his "Concrete Notes".[5]

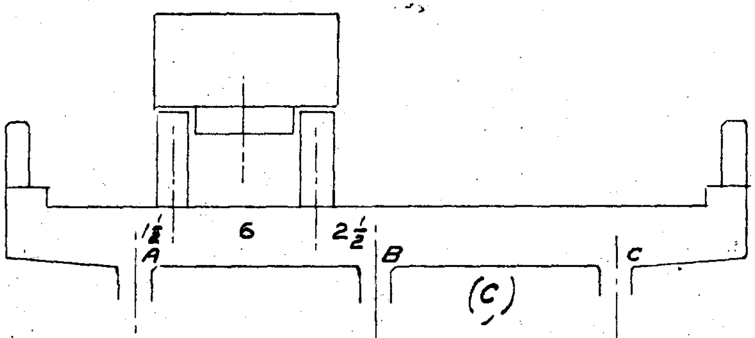
[5] Op.Cit.



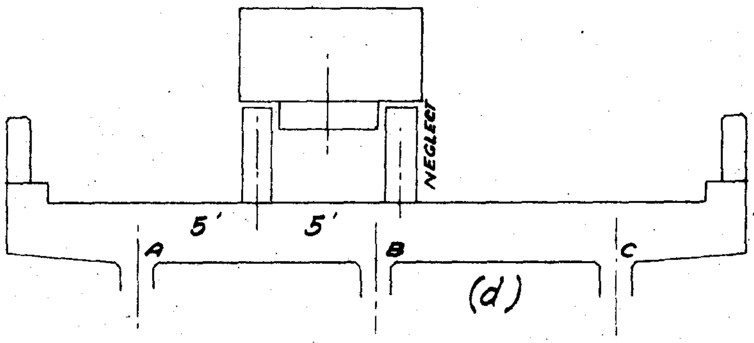
POSITION OF LOADS FOR MAX (−) MOM AT "B," ALSO FOR MAX LIVE LOAD ON ARCH RIB.



POSITION OF LOADS FOR STUDY OF INTERMEDIATE VALUES.

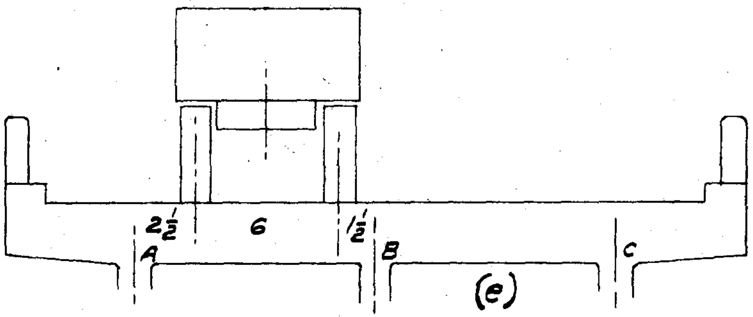


MAX + MOM AT QUARTER POINT OF GIRDER

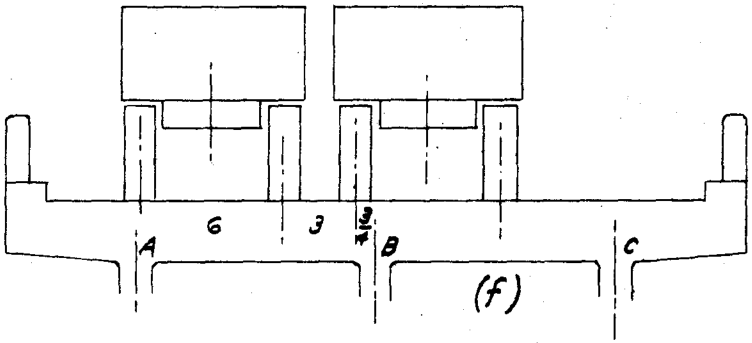


MAX + MOM IN CENTER OF GIRDER

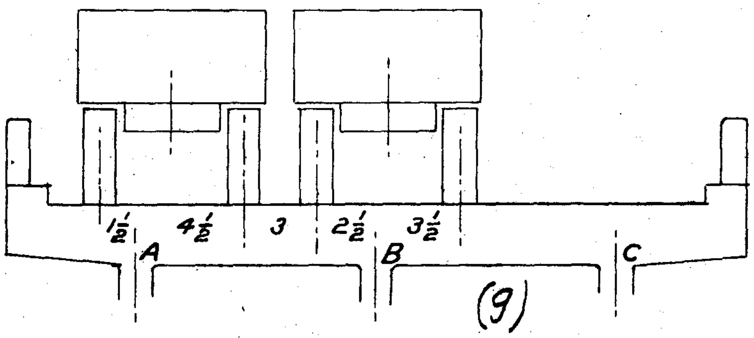
FIG 12 a-b-c-d



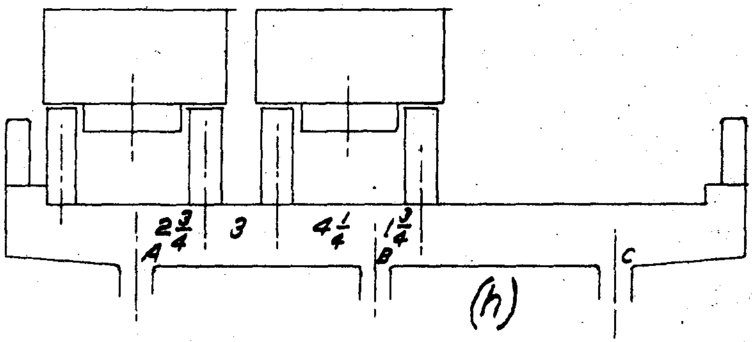
MAX + MOM AT LEFT QUARTER POINT OF GIRDER



MAXIMUM SHEAR AT THE CENTER COLUMN

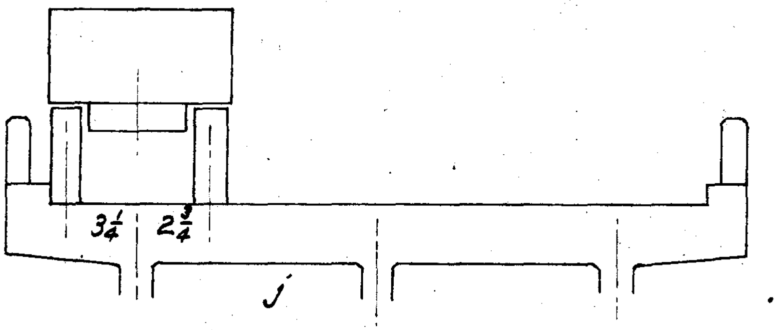


HIGH VALUE OF SHEAR AT RIGHT QUARTER POINT

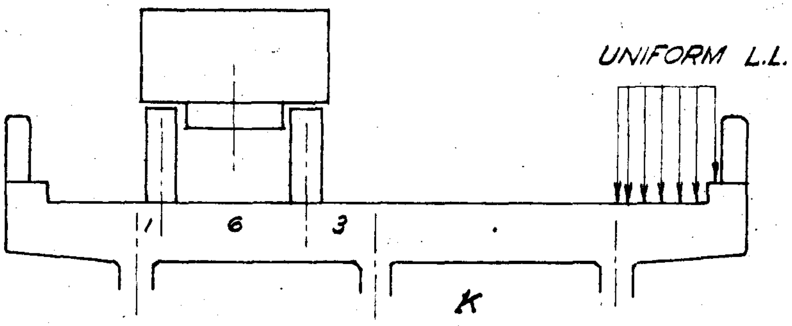


MAX SHEAR AT 2.75 FT FROM LEFT COLUMN.

FIG 12 e-f-g-h

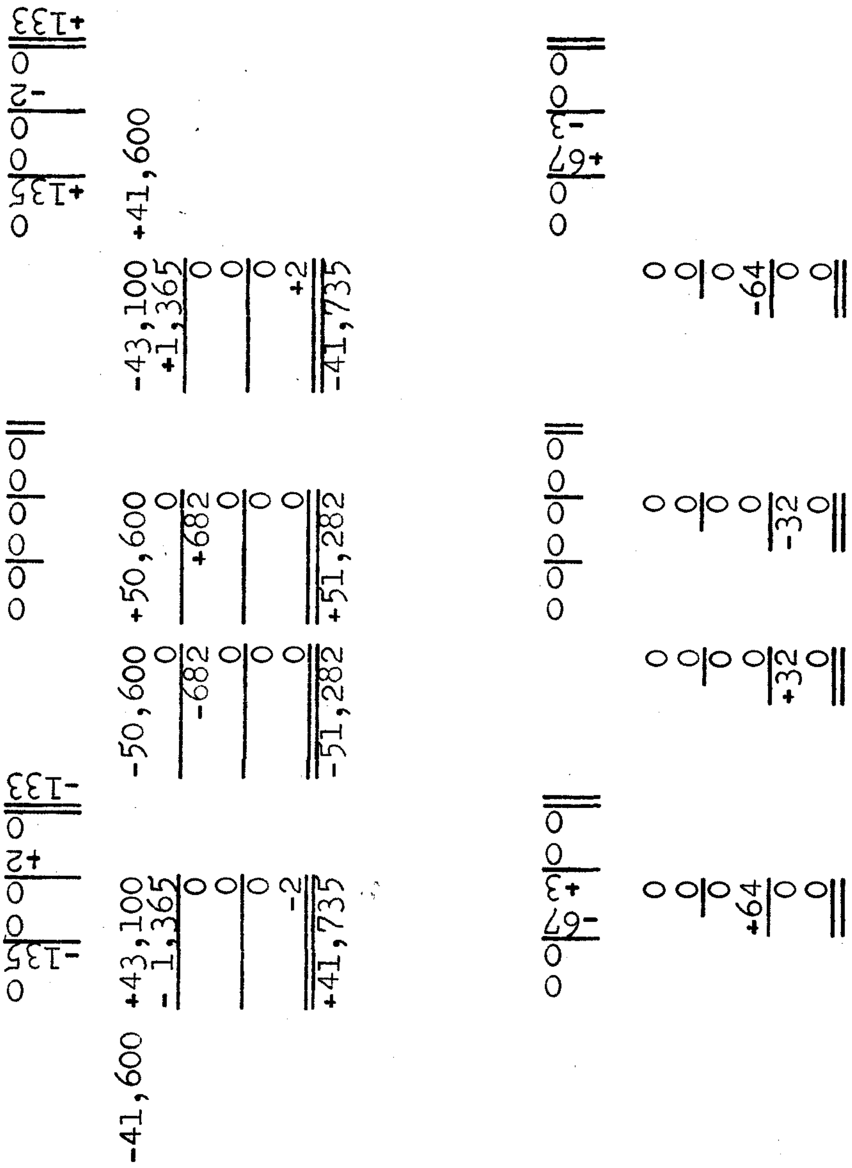


MAX NEG MOMENT AT $1\frac{1}{2}$ FT FROM AN OUTSIDE COLUMN.



MAX NEG MOMENT IN GIRDER

FIG 12 j-k

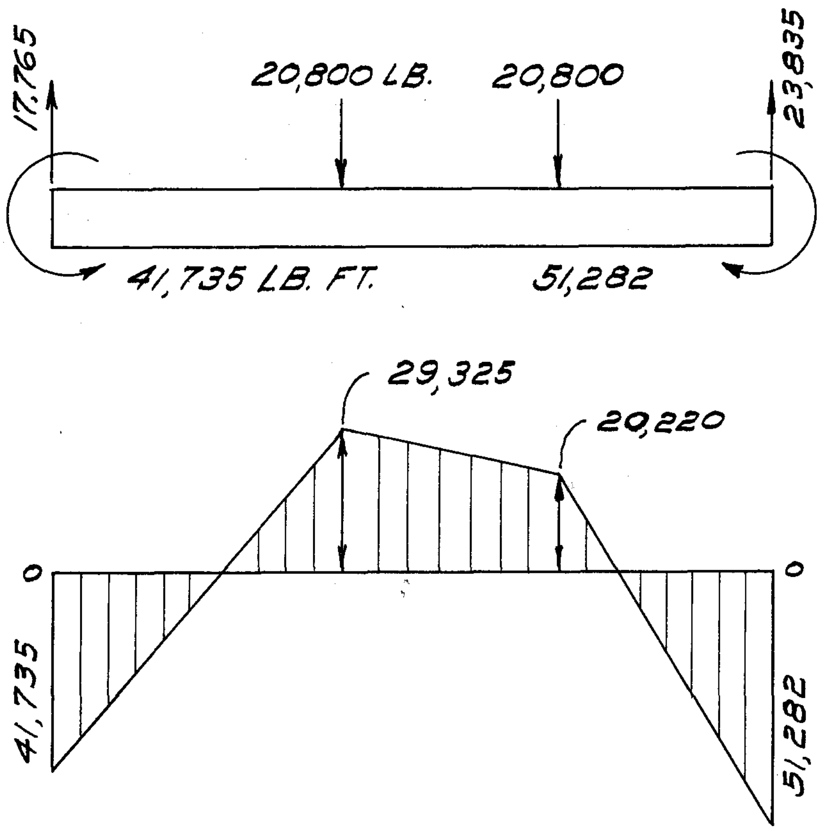


Maximum Negative Moment Over Center Column

$$MF_{AB} = 20{,}800 \times 4 \times (.6)^2 + 20{,}800 \times 7 \times (.3)^2 = 43{,}100 \text{ lb Ft}$$

$$MF_{BA} = 20{,}800 \times 3 \times (.7)^2 + 20{,}800 \times 6 \times (.4)^2 = 50{,}600 \text{ lb Ft}$$

No correction needed for sidesway



000	000	000
+427	-258	+616
000	000	000
+110	-265	+110
-4	+2	-6
+113	-56	+113
+646	-577	+833

-52,000	+47,260	-47,260	+52,400	-38,050	+31,200
	+4,313	-2,441	-2,441	+6,234	
	-1,220	+2,156	+3,117	-1,220	
	+1,110	-2,504	-2,504	+1,110	
	-1,252	+555	+555	-1,252	
	+1,143	-528	-528	+1,145	
	+51,354	-50,022	+50,599	-32,033	

000	000	000
000	000	000
+223	-129	+308
-9	+3	-12
+55	-132	+55
-4	+7	-4
+265	-251	+347

000	000	000	000
000	000	000	000
000	000	000	000
-214	+63	+63	-296
+31	-107	-148	+31
-82	+190	+190	-82
-265	+146	+105	-347

Study Of Intermediate Values

$MF_{AB} = 20{,}800 \times 3.5 \times (.65) + 20{,}800 \times 6.5 \times (.35) = 47{,}260$

MF_{BA} = Same as MF_{AB}

$MF_{BC} = 20{,}800 \times (2.5) \times (.75)^2 + 20{,}800 \times 5.5 \times (.45)^2 = 52{,}400$

$MF_{CB} = 20{,}800 \times 4.5 \times (.55)^2 + 20{,}800 \times 7.5 \times (.25)^2 = 38{,}050$

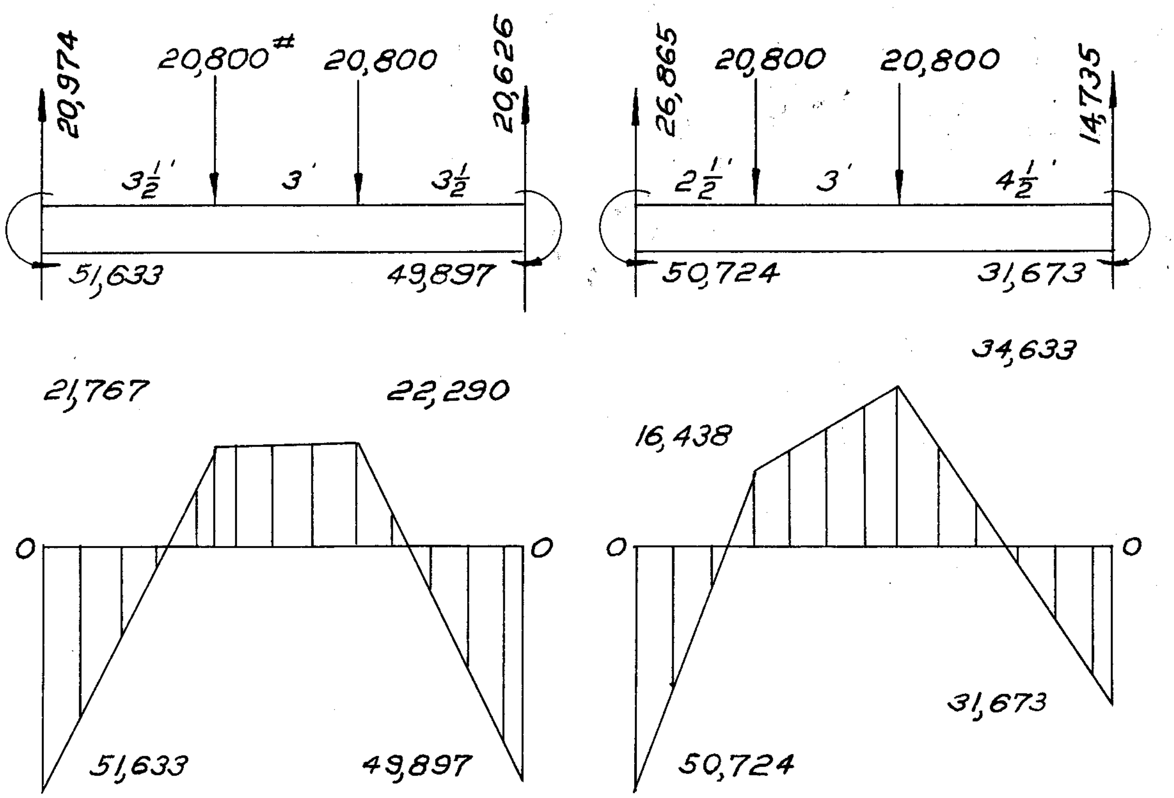
Correction for Sidesway

Unit correction = (-) algebraic sum of moments at ends of columns divided by arithmetic sum of the same.

$$= (-)\frac{1263}{2919} = -.43268$$

$M_{AB} = 51{,}354 + 279 = 51{,}633$ Moment Distribution

$M_{BA} = (-)\ 49{,}897 + 125 = (-)\ 49{,}772$ Signs



0 000
0 000
0 000
- 711
0 000
- 315
-1,026

0 000
0 000
0 000
0 000
- 355
+ 22
- 333

0 000
0 000
+7,898
-7,187
+3,500
-3,185
+1,026

0 000
0 000
0 000
0 000
- 203
+ 536
+ 333

0 000
+1,660
0 000
+ 740
- 8
+ 360
+2,752

0 000
0 000
+ 830
- 16
+ 370
- 21
+1,163

0 000
+15,795
0 000
+ 7,000
- 3,593
+ 3,417
+22,619

0 000
0 000
0 000
- 407
0 000
- 525
- 932

-33,250
+15,795
-14,740
+ 7,000
- 3,593
+ 3,417
-25,371

0 000
0 000
0 000
- 407
- 701
- 525
- 231

0 000
-2,920
0 000
- 711
+ 29
- 318
-3,920

0 000
0 000
-1,460
+ 58
- 355
+ 22
-1,735

+32,400
-29,480
+ 7,898
- 7,187
+ 3,500
- 3,211
+ 3,920

0 000
0 000
0 000
+1,402
- 203
+ 536
+1,735

Maximum Positive Moment At Right Quarter Point

MF_{AB} = 20,800 x 1.5 x $(.85)^2$ 20,800 x 7.5 x $(.25)^2$

= 32,400. lb ft.

MF_{BA} = 20,800 x 2.5 x $(.75)^2$ 20,800 x 8.5 x $(.15)^2$

= 33,250. lb ft.

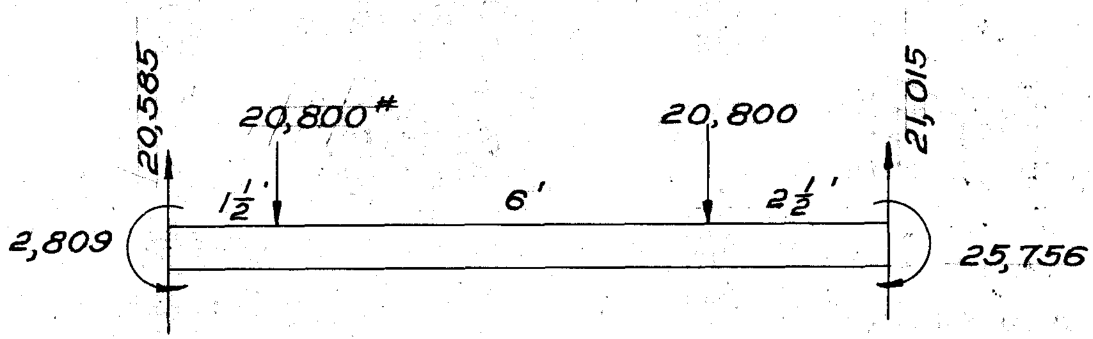
Correction for Side Sway

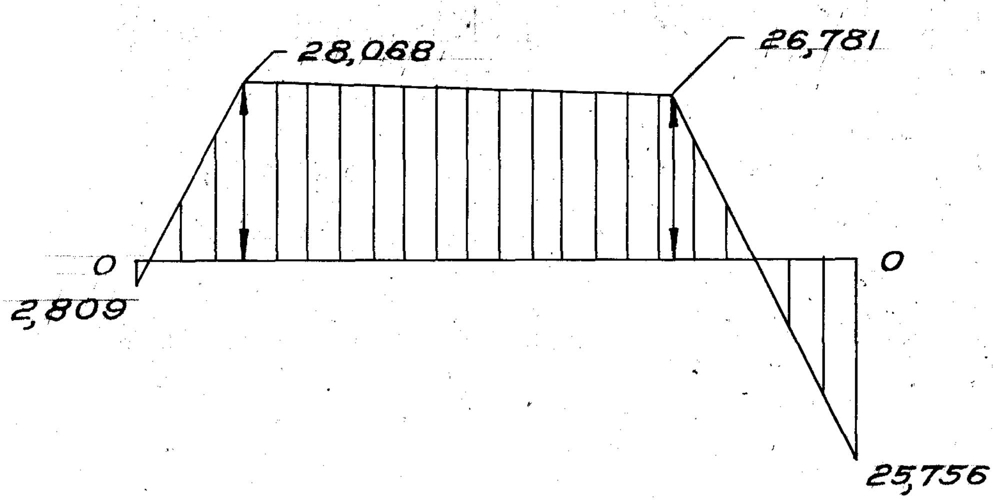
Unit correction = (-) algebraic sum of moments at ends of columns divided by arithmetic sum of the same.

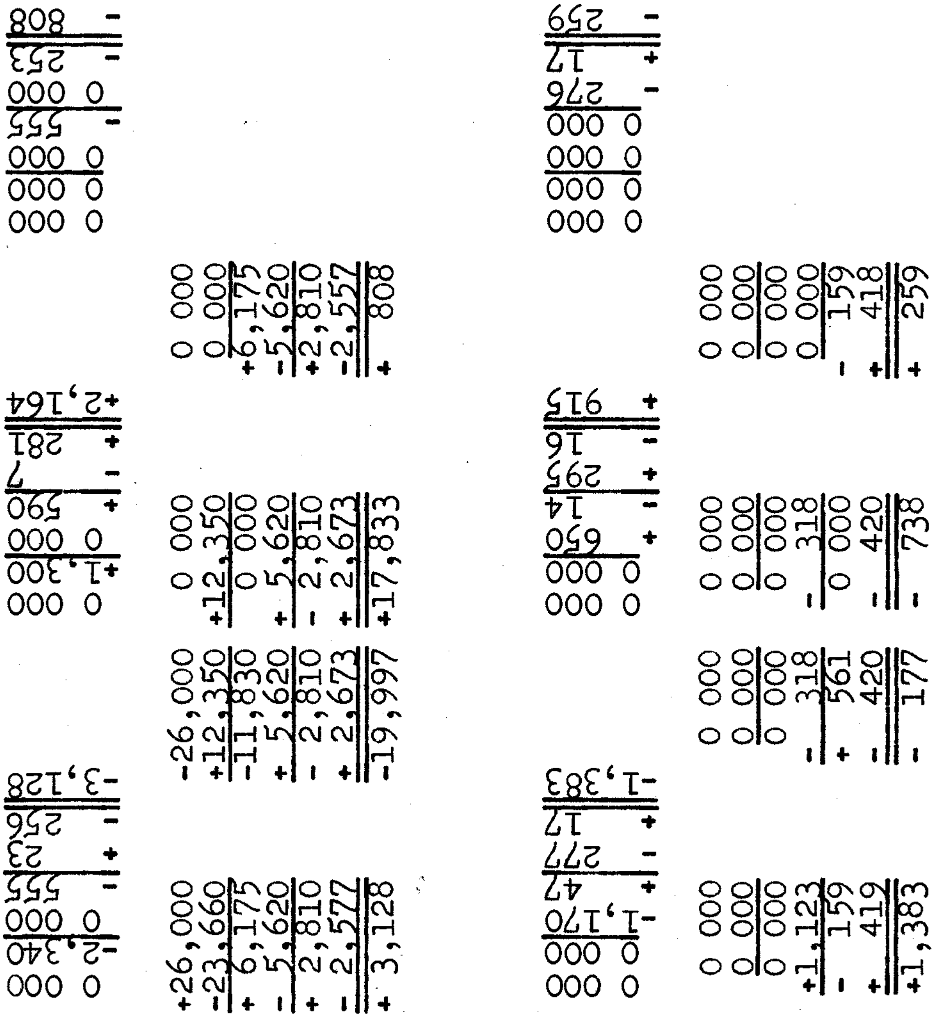
$$= \frac{3099}{10929} = .2835$$

M_{AB} = 3920 - 1111 = 2809 Moment Distribution

M_{BA} = -25371 - 385 = -25756 Signs







Maximum Positive Moment At Center Of Girder

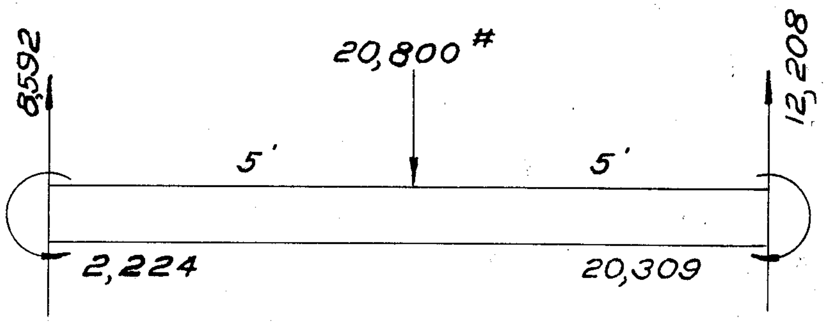
$$MF_{AB} = 20{,}800 \times 5 \times (.5)^2 = 26{,}000 \text{ lb ft.}$$

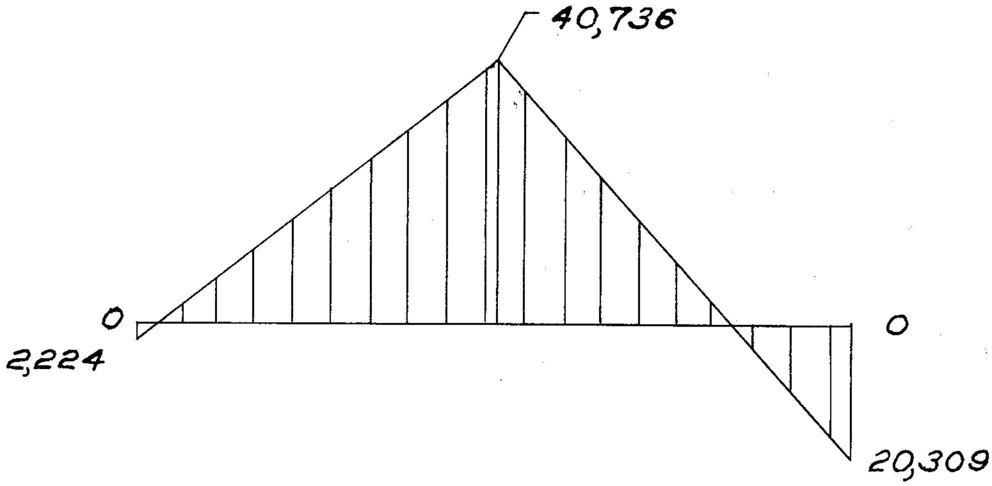
Correction for Side Sway

$$\text{Unit correction} = \frac{-2500}{8657} = -.2885$$

$M_{AB} = 3128 - 904 = 2224$ Moment Distribution Signs

$M_{BC} = -19997 - 312 = -20309$





0 000	0 000	0 000		
-3,000	+1,620	0 000		
0 000	0 000	0 000		
- 695	+ 755	- 695		
+ 30	- 8	0 000		
- 326	+ 350	- 323		
-3,991	+2,717	-1,018		

0 000	+33,250	-32,400	0 000	0 000
	-30,250	+15,390	+15,390	0 000
	+ 7,695	-15,125	0 000	+7,695
	- 7,000	+ 7,185	+ 7,185	-7,000
	+ 3,592	- 3,500	- 3,500	+3,592
	- 3,296	+ 3,329	+ 3,329	-3,269
	+ 3,991	-25,121	+22,404	+1,018

0 000	0 000	0 000
0 000	0 000	0 000
-1,500	+ 810	0 000
+ 60	- 16	0 000
- 348	+ 378	- 348
+ 22	- 22	+ 22
-1,766	+1,150	- 326

0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
+1,440	- 397	- 397	0 000
- 198	+ 720	0 000	- 198
+ 524	- 538	- 538	+ 524
+1,766	- 215	- 935	+ 326

Maximum Positive Moment At Left Quarter Point

$$MF_{AB} = 20,800 \times 2.5 \times (.75)^2 + 20,800 \times 8.5 \times (.15)^2$$
$$= 33250 \text{ lb ft.}$$

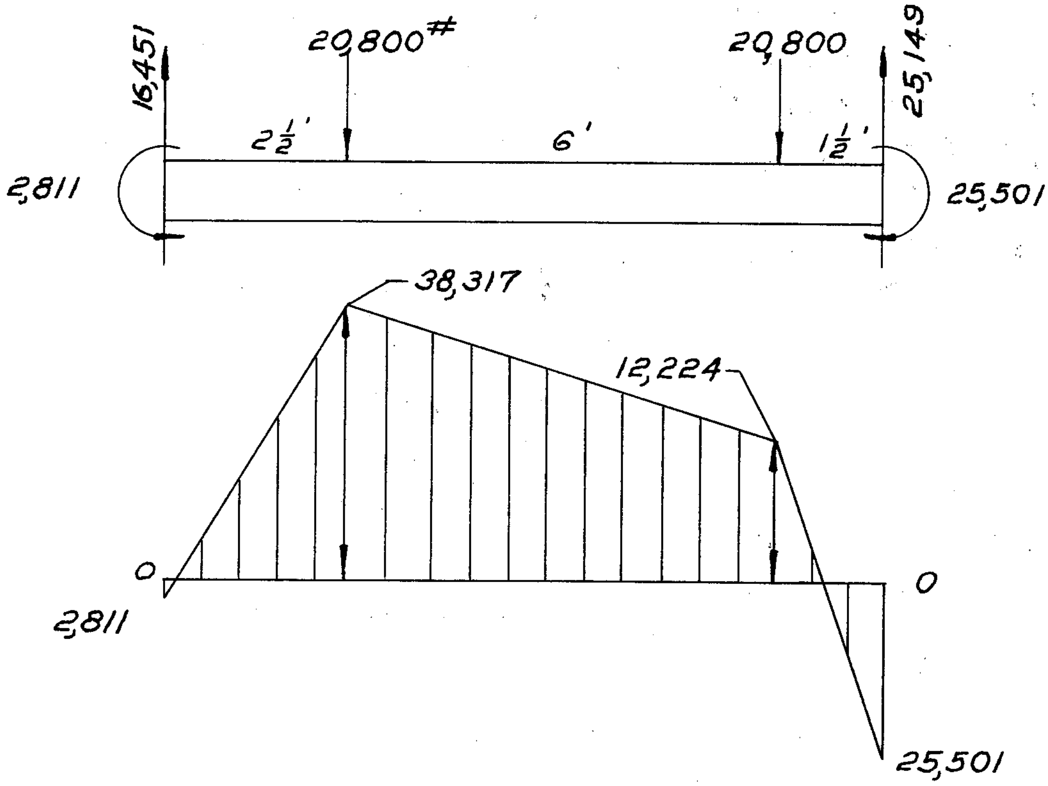
$$MF_{BA} = 20,800 \times 1.5 \times (.85)^2 + 20,800 \times 7.5 \times (.25)^2$$
$$= 32,400 \text{ lb ft.}$$

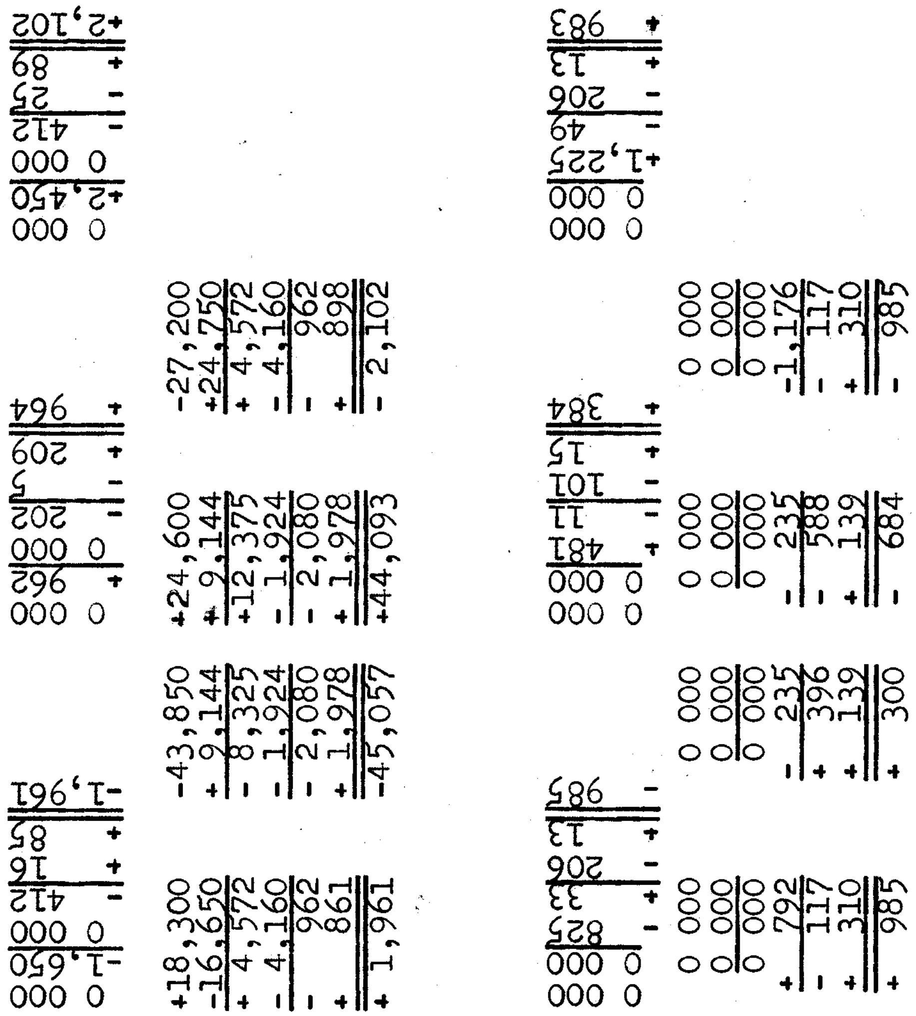
Correction for Side Sway

Unit correction $= \frac{3234}{10968} = .295$

$M_{AB} = 3991 - 1180 = 2811$ Moment Distribution Signs

$M_{BA} = -25121 - 380 = -25501$





Maximum Shear At Center Column Support

$$MF_{AB} = 20{,}800 \times 6.25 \times (.375)^2 = 18{,}300$$

$$MF_{BA} = 20{,}800 \times 3.75 + 20{,}800 \times 7.5 \times (.925)^2 = 43{,}850$$

$$MF_{BC} = 20{,}800 \times 5.25 \times (.475)^2 = 24{,}600$$

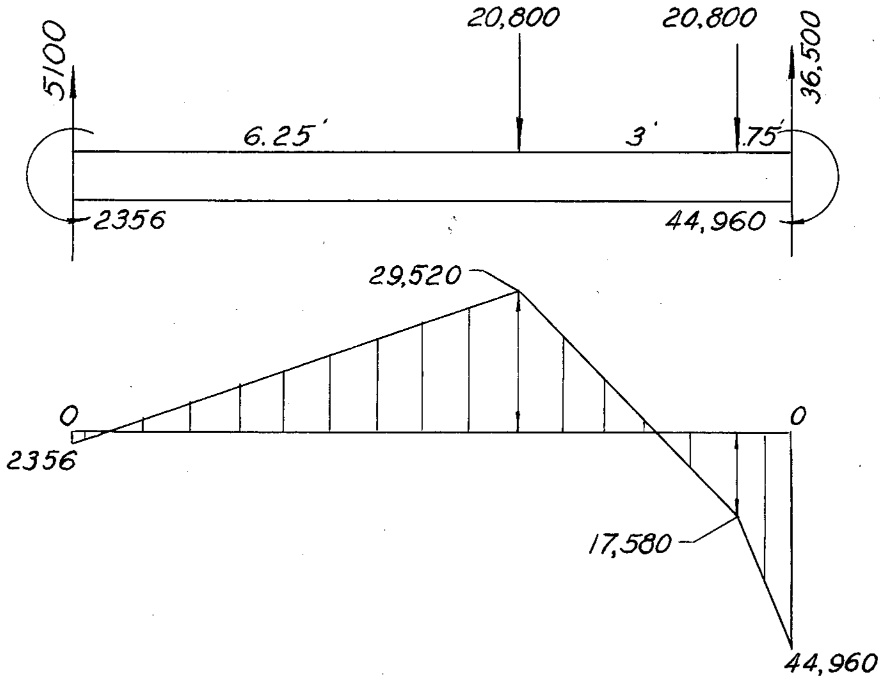
$$MF_{CB} = 20{,}800 \times 4.75 \times (.525)^2 = 27{,}200$$

Correction for Side Sway

$$\text{Unit correction} = \frac{1487}{7379} = .2015$$

$$M_{AB} = 1961 + 395 = +2356$$

$$M_{BA} = -45057 + 97 = -44960$$



-31,200	+38,050	-52,400	+30,800	-16,600
	- 6,234	+10,260	+10,260	+15,100
	+ 5,130	- 3,117	+ 7,550	+ 5,130
	- 4,668	- 2,106	- 2,106	- 4,668
	- 1,053	- 2,334	- 2,334	- 1,053
	+ 953	+ 2,220	+ 2,220	+ 972
	+32,178	-47,477	+46,390	- 1,119

0 000	0 000	0 000
- 616	+1,080	+1,500
0 000	0 000	0 000
- 462	- 221	- 462
+ 6	- 5	- 15
+ 94	+ 233	+ 96
- 978	+1,087	+1,119

0 000	0 000	0 000
0 000	0 000	0 000
- 308	+ 540	+ 750
+ 12	- 10	- 30
- 231	- 110	- 231
+ 14	+ 6	+ 14
- 513	+ 426	+ 503

0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
+ 296	- 265	- 265	- 720
- 132	+ 148	- 360	- 132
+ 349	+ 158	+ 158	+ 349
+ 513	+ 41	- 467	- 503

Maximum Shear At Right Quarter Point

$$MF_{AB} = 20{,}800 \times 4.5 \times (.55)^2 + 20{,}800 \times 7.5 \times (.25)^2 = 38050$$

$$MF_{BA} = 20{,}800 \times 5.5 \times (.45)^2 + 20{,}800 \times 2.5 \times (.75)^2 = 52400$$

$$MF_{BC} = 20{,}800 \times 3.5 \times (.65)^2 = 30{,}800$$

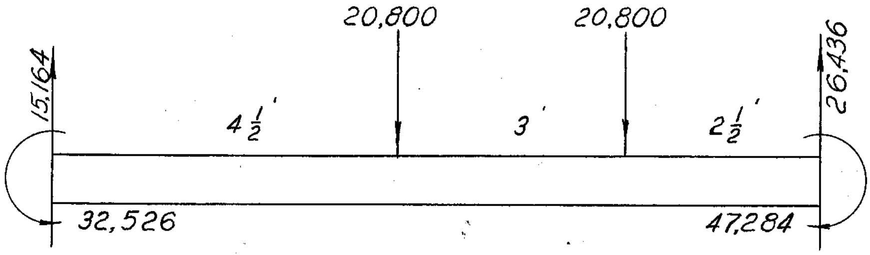
$$MF_{CB} = 20{,}800 \times 6.5 \times (.35)^2 = 16{,}600$$

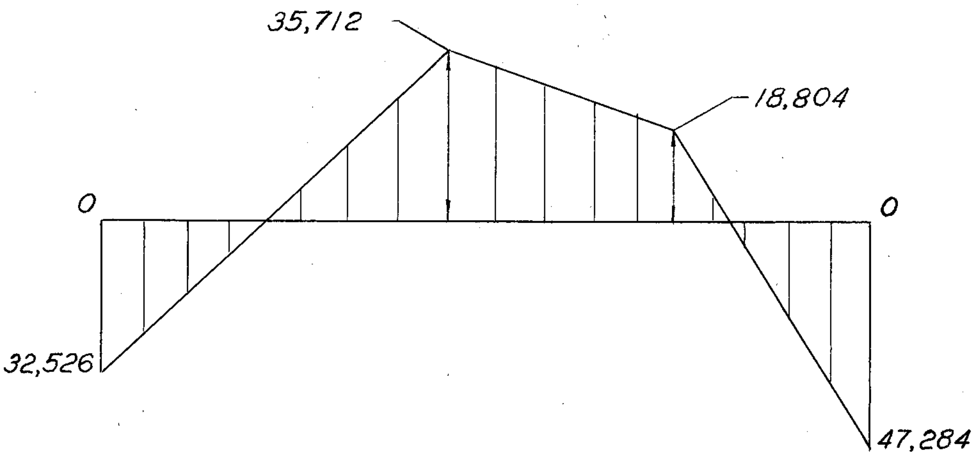
Correction for Side Sway

Unit correction $= \frac{1644}{4616} = .3565$

$M_{AB} = +32178 + 348 = 32{,}526$ Moment Distribution

$M_{BA} = -47477 + 193 = -47{,}284$ Signs





0 000	0 000	0 000	
+1,430	+800	+472	
0 000	0 000	0 000	
-340	-480	-339	
-15	-4	-5	
+207	+171	+206	
+1,282	+487	+334	

-67,600	+51,700	-40,700	+24,800	-5,250
	+14,470	+ 7,550	+ 7,550	+4,778
	+ 3,775	+ 7,235	+ 2,389	+3,775
	- 3,435	- 4,572	- 4,572	-3,436
	- 2,286	- 1,717	- 1,718	-2,286
	+ 2,094	+ 1,634	+ 1,634	+2,085
	+66,318	-30,570	+30,083	- 334

0 000	0 000	0 000
0 000	0 000	0 000
+715	+400	+236
-30	-8	-10
-170	-240	-170
+11	+15	+11
+526	+167	+67

0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
- 685	- 196	- 196	- 226
- 98	- 342	- 113	- 98
+ 257	+ 340	+ 340	+ 257
- 526	- 198	+ 31	- 67

Maximum Shear At 2.75 Feet From Outer Column

$$MF_{AB} = 20{,}800 \times 2.75 \times (.725)^2 + 20{,}800 \times 5.75 \times (.425)^2$$
$$= 51{,}700$$

$$MF_{BA} = 20{,}800 \times 4.25 \times (.575)^2 + 20{,}800 \times .725 \times (.275)^2$$
$$= 40{,}700$$

$$MF_{BC} = 20{,}800 \times 1.75 \times (.825)^2 = 24{,}800$$

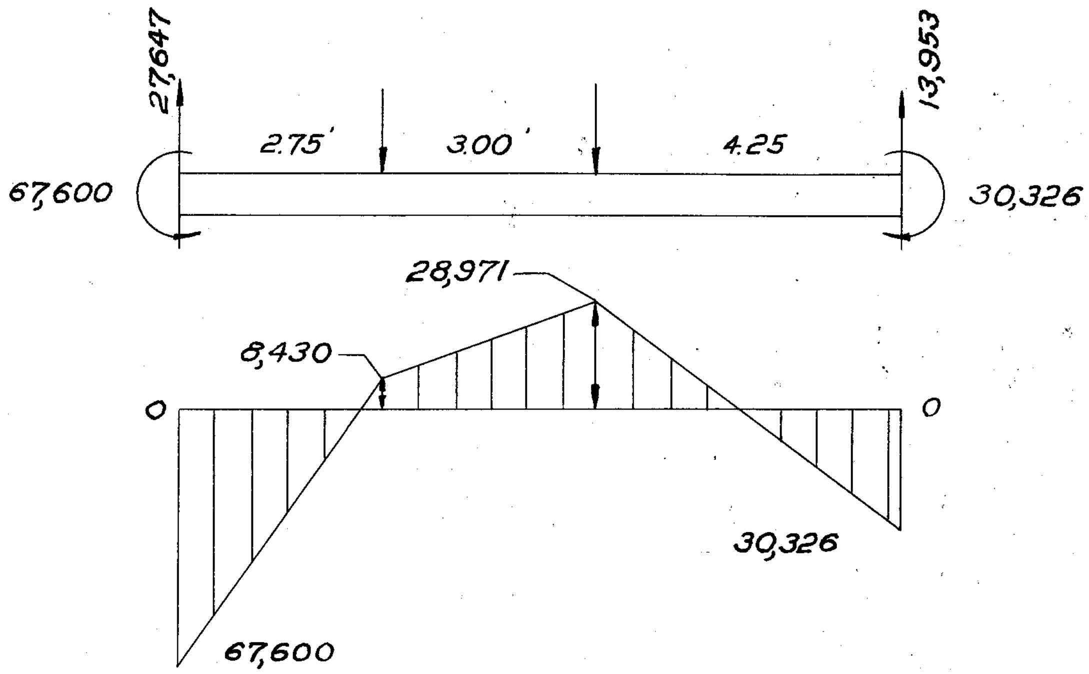
$$MF_{CB} = 20{,}800 \times 8.25 \times (.175)^2 = 5{,}250$$

Correction for Side Sway

Unit correction $= \frac{28630}{28630} = 1.00$

$M_{AB} = 66{,}318 + 1{,}282 = 67{,}600$ Moment Distribution

$M_{BA} = -30{,}570 + 244 = -30{,}326$ Signs



-20,800

+32,650	-52,600	+30,000	-20,000
-10,780	+10,735	+10,735	+18,200
+ 5,368	- 5,390	+ 9,100	+ 5,368
- 4,885	- 1,762	- 1,762	- 4,885
- 881	- 2,442	- 2,442	- 881
+ 792	+ 2,323	+ 2,323	+ 818
+22,254	-49,136	+47,954	- 1,390

0 000	0 000	0 000
-1,070	+1,130	+1,800
0 000	0 000	0 000
- 473	- 186	- 473
+ 10	- 6	- 18
+ 78	+ 244	+ 81
-1,455	+1,182	+1,390

0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
0 000	0 000	0 000	0 000
+ 514	- 276	- 276	- 864
- 138	+ 257	- 432	- 138
+ 359	+ 131	+ 131	+ 359
+ 735	+ 112	- 577	- 643

0 000	0 000	0 000
0 000	0 000	0 000
- 535	+ 565	+ 900
+ 21	- 13	- 36
- 236	- 93	- 236
+ 15	+ 6	+ 15
- 735	+ 465	+ 643

Study For Shear At Center Of Girder

$$MF_{AB} = 20{,}800 \times 5 \times (.50)^2 + 20{,}800 \times 8 \times (.2)^2 = 32{,}650$$

$$MF_{BA} = 20{,}800 \times 5 \times (.50)^2 + 20{,}800 \times 2 \times (.8)^2 = 52{,}600$$

$$MF_{BC} = 20{,}800 \times 4 \times (.6)^2 = 30{,}000$$

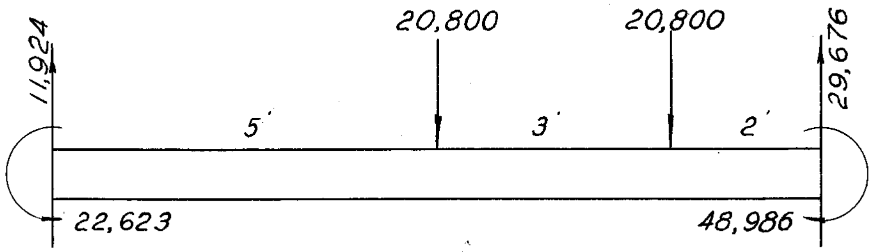
$$MF_{CB} = 20{,}800 \times 6 \times (.4)^2 = 20{,}000$$

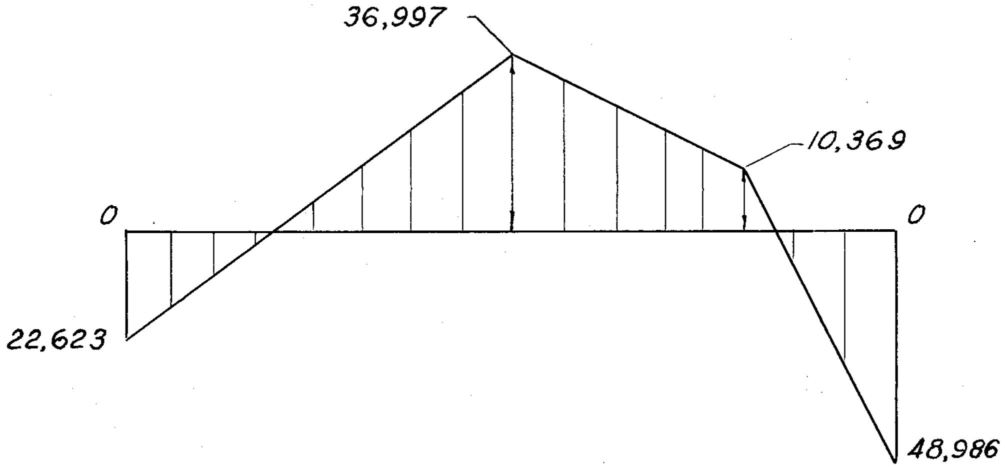
Correction For Side Sway

Unit correction $= \frac{1490}{5870} = .254$

$M_{AB} = 22{,}254 + 369 = 22{,}623$ Moment Distribution

$M_{BA} = -49{,}136 + 150 = -48{,}986$ Signs





0 000	0 000	0 000
+3,375	+670	0
0	0	0
- 244	- 852	- 244
- 34	- 2	0
+ 368	+ 124	+ 364
+3,465	- 160	+ 120

-67,600	+30,100	-11,400	0 000	0 000
	+34,125	+ 5,415	+5,415	0
	+ 2,708	+17,062	0	+2,708
	-2,464	-8,105	-8,105	-2,464
	-4,052	-1,232	-1,232	-4,052
	+ 3,718	+ 1,171	+1,171	+3,680
	+64,135	+ 2,911	-2,751	- 128

0 000	0 000	0 000
0	0	0
+1,688	+ 285	0
- 68	- 5	0
- 122	- 426	- 122
+ 8	+ 25	+ 8
+1,500	- 121	- 114

0,000	0 000	0 000	0 000
0	0	0	0
0	0	0	0
-1,614	- 140	- 140	0
- 70	- 807	0	- 70
+ 184	+ 604	+ 604	+ 184
-1,500	- 343	+464	+ 114

Maximum Negative Moment at $1\frac{1}{2}$ Feet from Outside Column

$MF_{AB} = 20{,}800 \times 2.75 \times (.725)^2 = 30{,}100$ lb ft.

$MF_{BA} = 20{,}800 \times 7.25 \times (.275)^2 = 11{,}400$ lb ft.

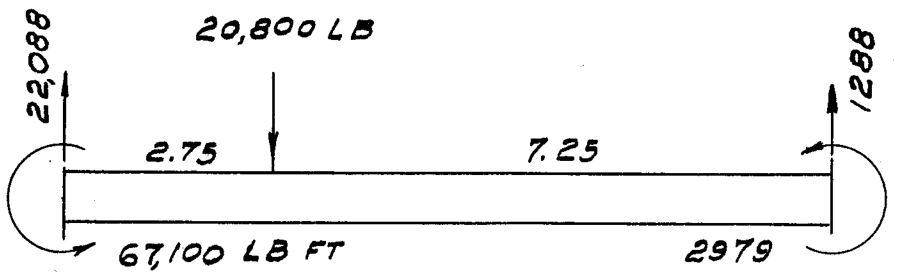
Cantilever Moment = 20,800 x 3.25 = 67,600 lb ft.

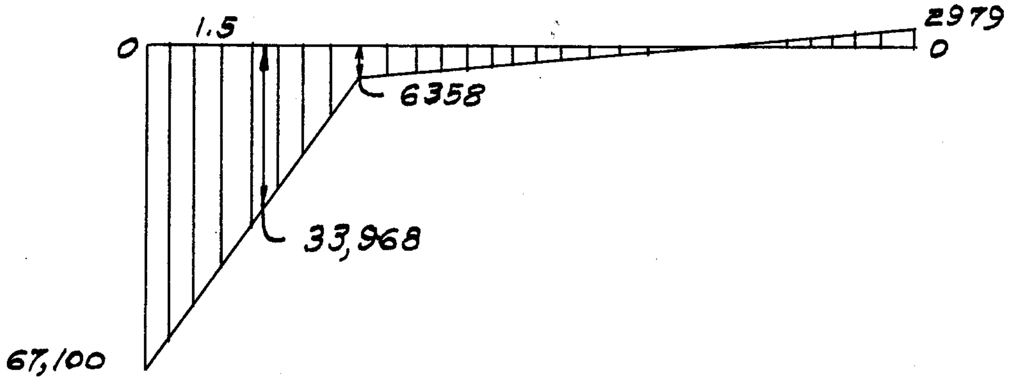
Correction for sidesway

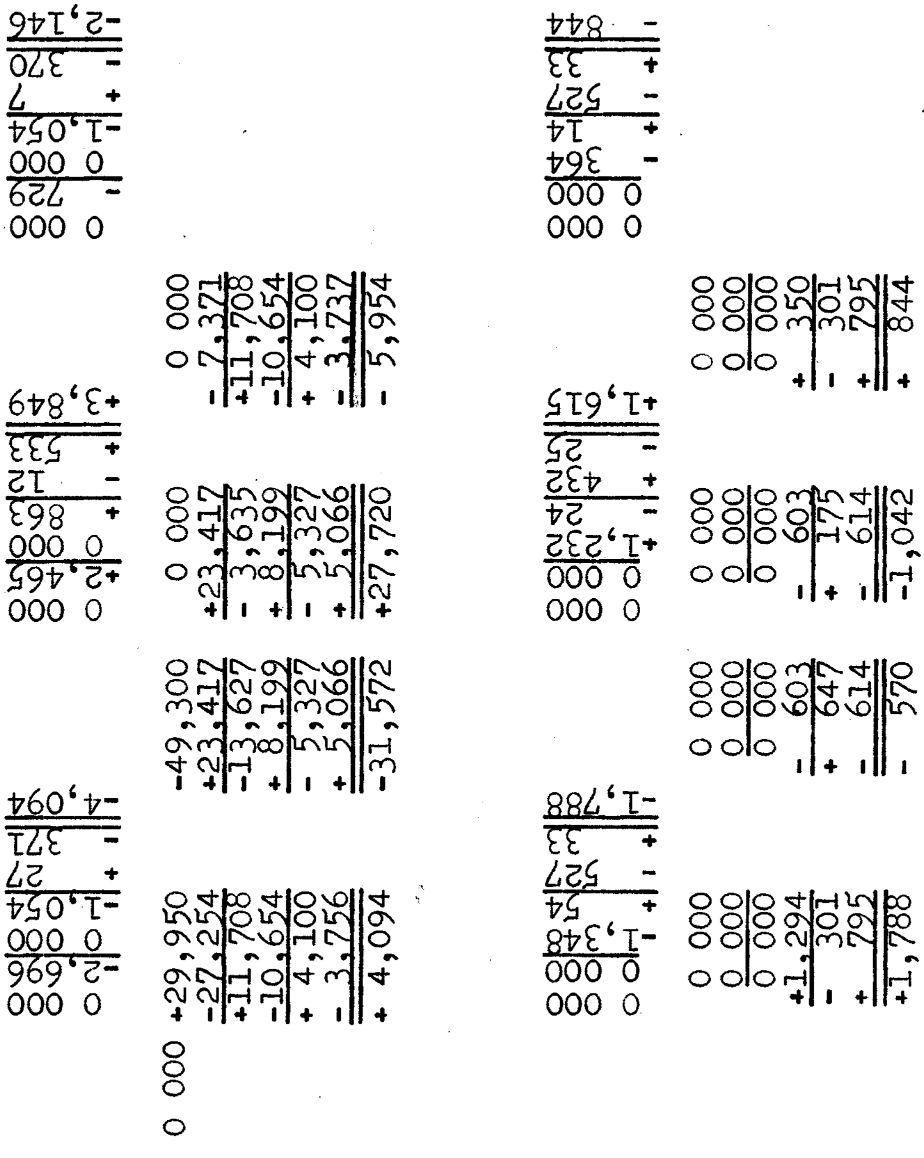
$$\text{Unit correction} = \frac{4690}{5480} = .856$$

M_{AB} = 64,135 + 2965 = 67,100 Moment Distribution Signs

M_{BA} = 2911 + 68 = 2,979







Maximum Negative Moment In Girder

$$MF_{AB} = 20,800 \times 1 \times (.9)^2 + 20,800 \times 7 \times (.3)^2 = 29,950$$

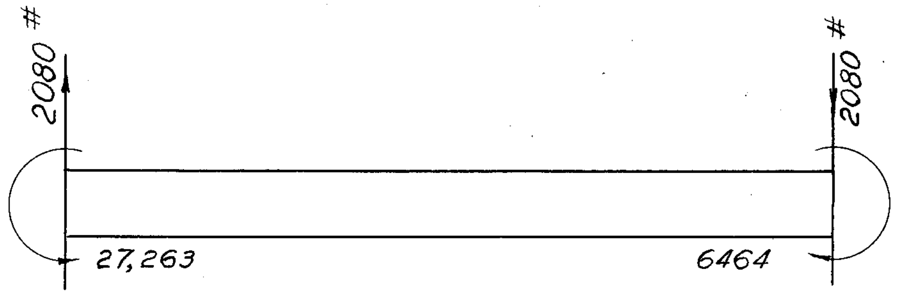
$$MF_{BA} = 20,800 \times 3 \times (.7)^2 + 20,800 \times 9 \times (.1)^2 = 49,300$$

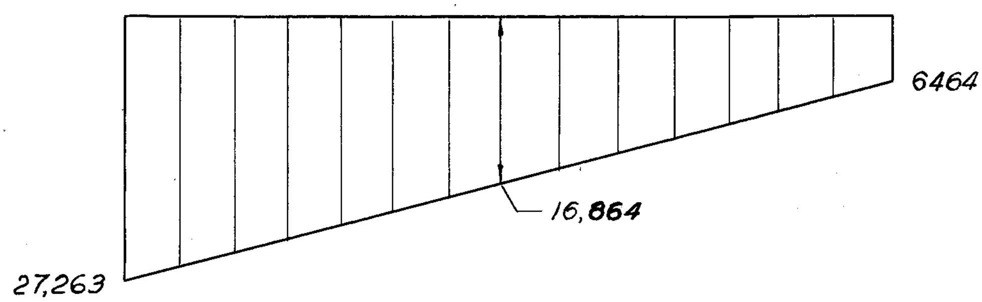
$$\text{Cantilever} = 4\tfrac{1}{2} \times 8 \times 100 \times 2\tfrac{1}{4} = 8,100$$

Correction For Side Sway

$M_{BC} = 27,720 - 457 = 27,263$ Moment Distribution

$M_{CB} = 5,954 + 510 = 6,464$ Signs





Because of the assumptions made regarding the relative deflections of the columns supporting the girders, it is not thought necessary to repeat the dead load computations for the short column bent, as a small change in the dead load moments and shears will be negligible in comparison with the total combined dead and live load moments and shears. However, the short column bent will be checked for live load moments and shears at certain critical points to see if there results any appreciable difference.

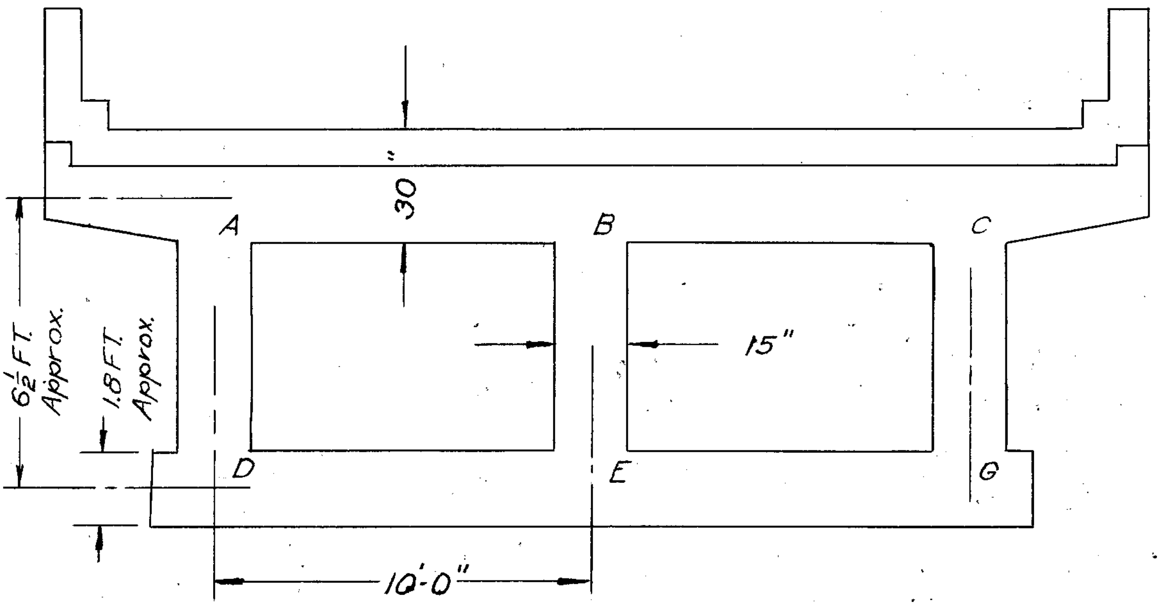
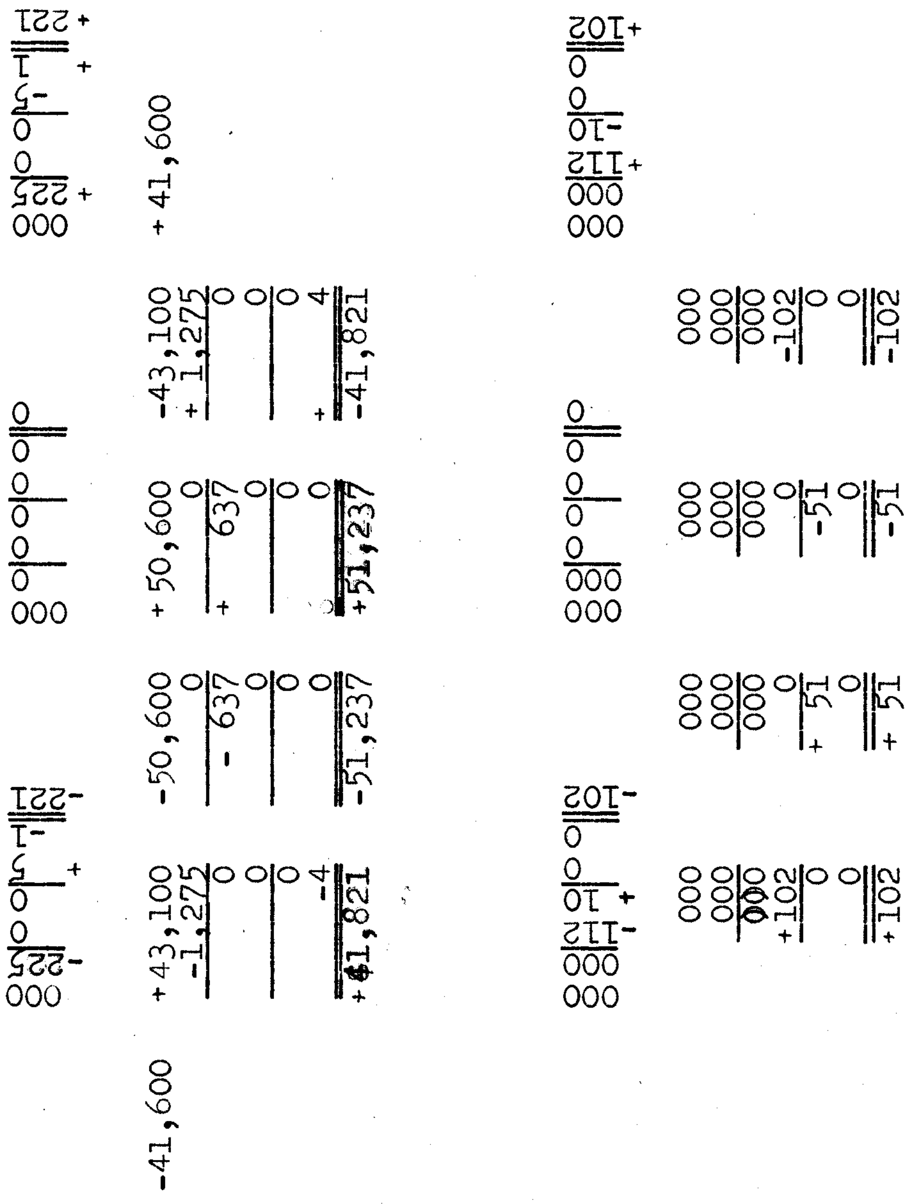
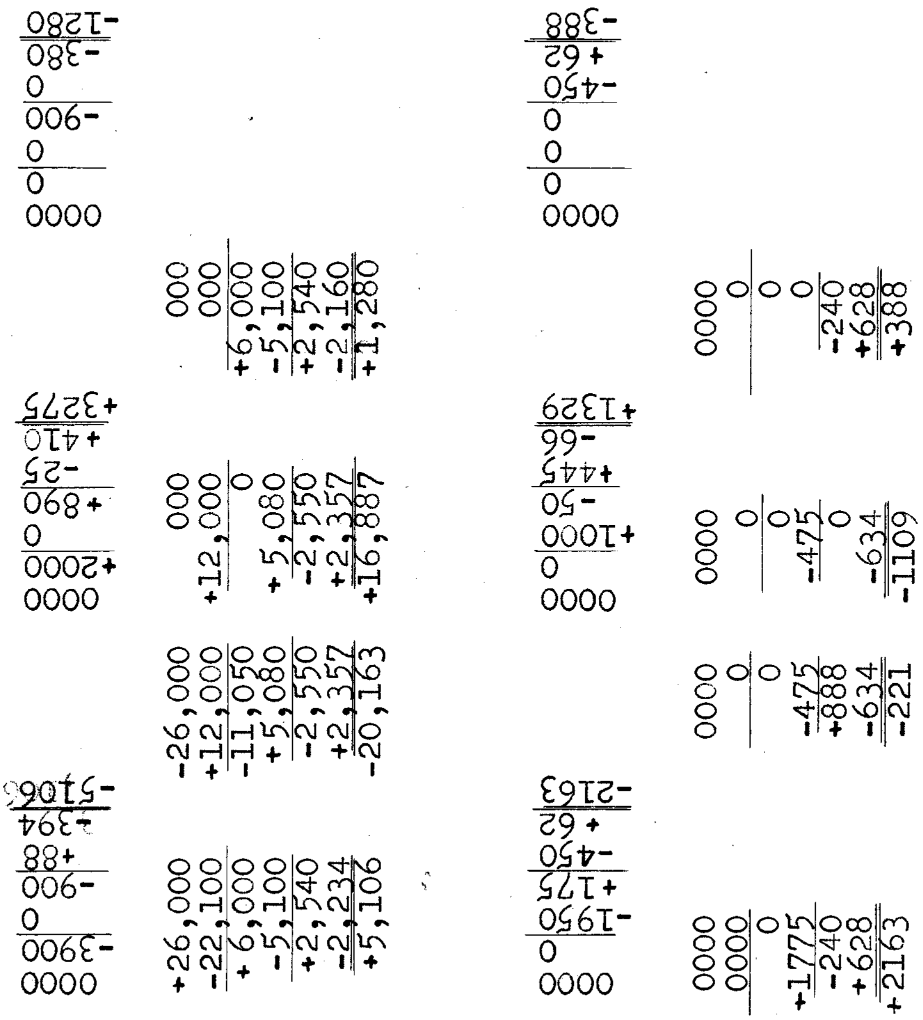


FIG 13



Maximum Negative Moment Over Center Column

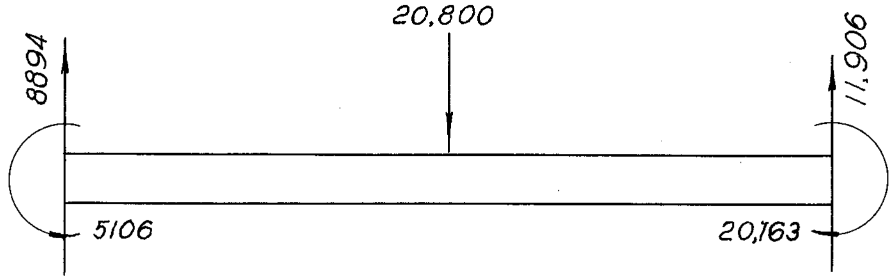


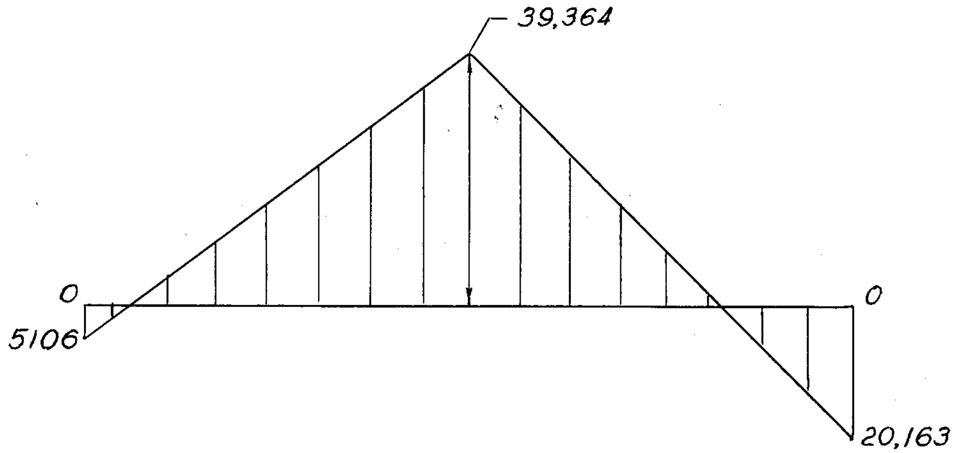
Maximum Positive Moment At Center Of Girder

Fixed-end moments are the same as for the long column bent.

Since this bent is so near the center of the arch where the roadway is fastened to the arch proper, we shall assume that no side sway is possible.

By comparison with the calculations for the long column bent, it may be seen that the maximum positive given here is slightly less.





0000		0000	0000
-2750		+1540	+4080
0		0	0
-664		-303	-664
+62		-19	-92
+121		+301	+145
-3231		+1519	+3469

+18,300	-43,850	+24,600	-27,200
-15,550	+8,855	+ 8,855	+23,120
+ 4,427	-7,775	+11,560	+4,427
-3,763	-1,741	-1,741	-3,763
-870	-1,881	-1,881	-870
+ 687	+1,740	+1,740	+817
+3,231	-44,652	+43,133	-3,469

0000		0000	0000
0		0	0
-1375		+ 770	+2040
+124		-38	-184
+332		-151	-332
+46		+ 22	+ 46
-1537		+603	+1570

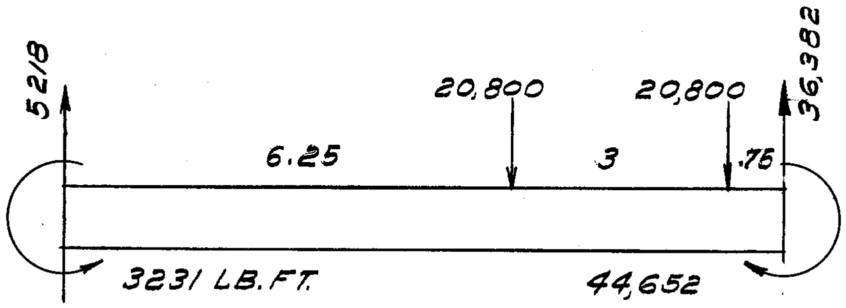
0000	0000	0000	0000
0	0	0	0
0	0	0	0
+1251	-366	-366	-1856
-183	+625	-928	-183
+469	+216	+216	+ 469
+ 1537	+ 475	-1078	-1570

Maximum Shear At Center Column Support

Fixed-end moments are the same as for the long column bent.

Because of the nearness to the crown, side sway is assumed to be impossible.

By comparison with the computations given for the long column bent it may be seen that there is a negligible difference of 118 lb in the shears.



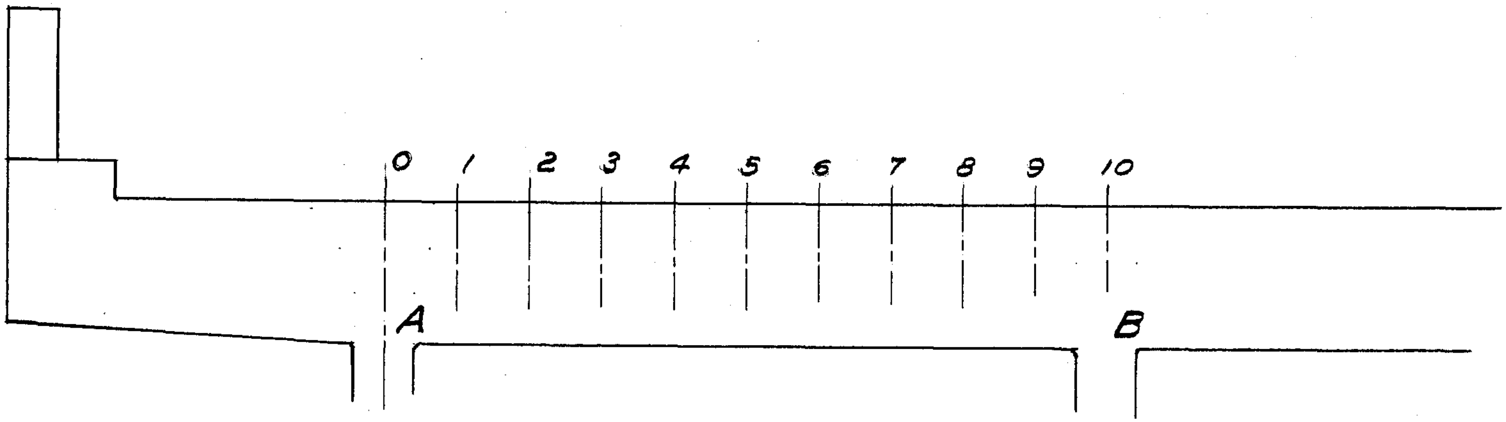


TABLE VII MAXIMUM GIRDER MOMENTS IN FT LBS

POINT	0 FT	$1\frac{1}{2}$	$2\frac{1}{2}$	5	$7\frac{1}{2}$	$8\frac{1}{2}$	10
DEAD LOAD	-31582	-17500	-10900	-5948	-11200	-16200	-26814
	-26078	-15800	- 9500	+2600	+ 4000	+ 2500	- 4715
LIVE LOAD NEG	-67600	-33968	-11664	-16840	-22064	-24143	-51282
LIVE LOAD POS		+28068	+38317	+40736	+26781	+12224	
MAXIMUM POS		+12268	+28718	+43336	+30781	+14724	
MAXIMUM NEG	-99182	-51468	-22564	-22788	-33264	-40343	-78096

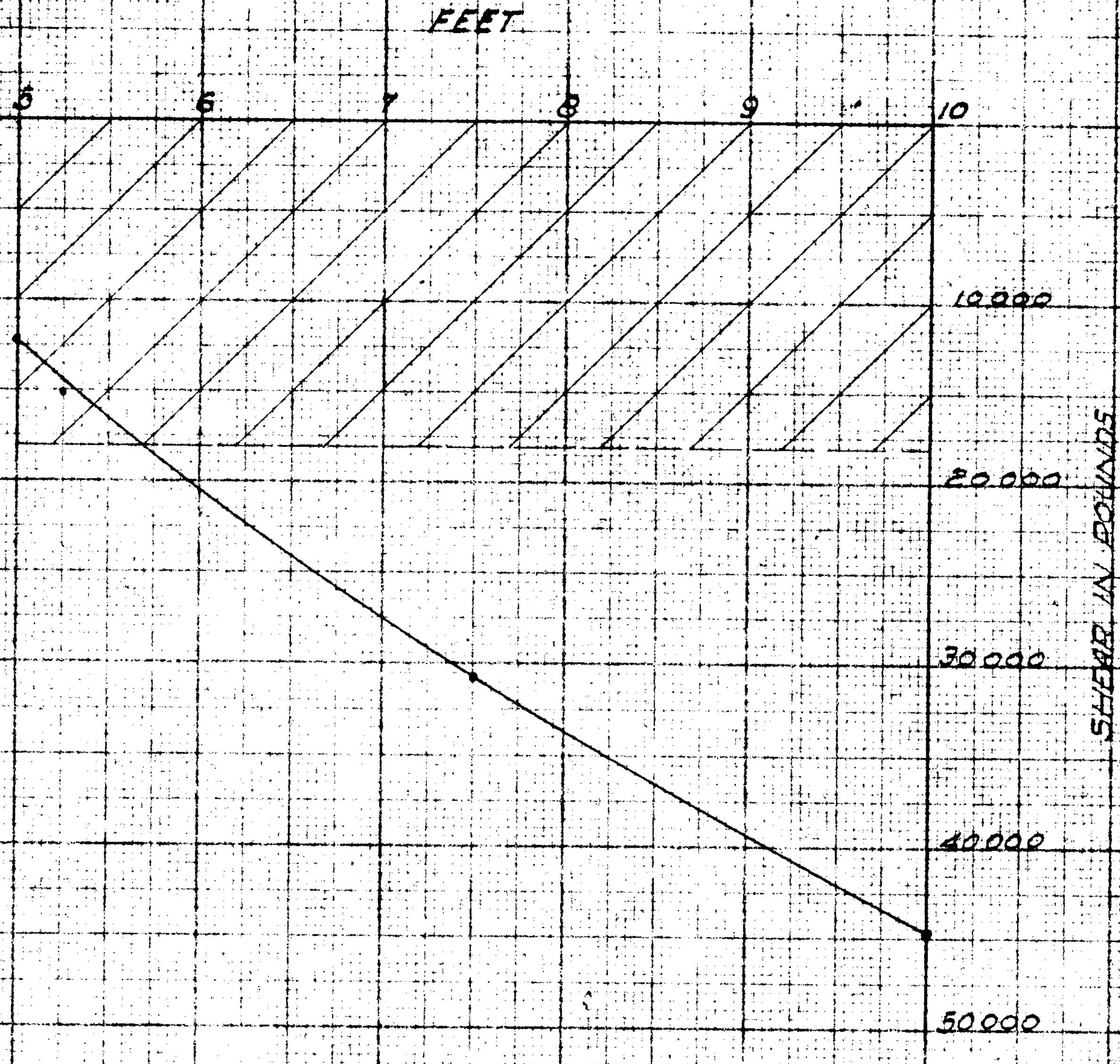


FIG 14

GIRDER
MAXIMUM SHEAR CURVE

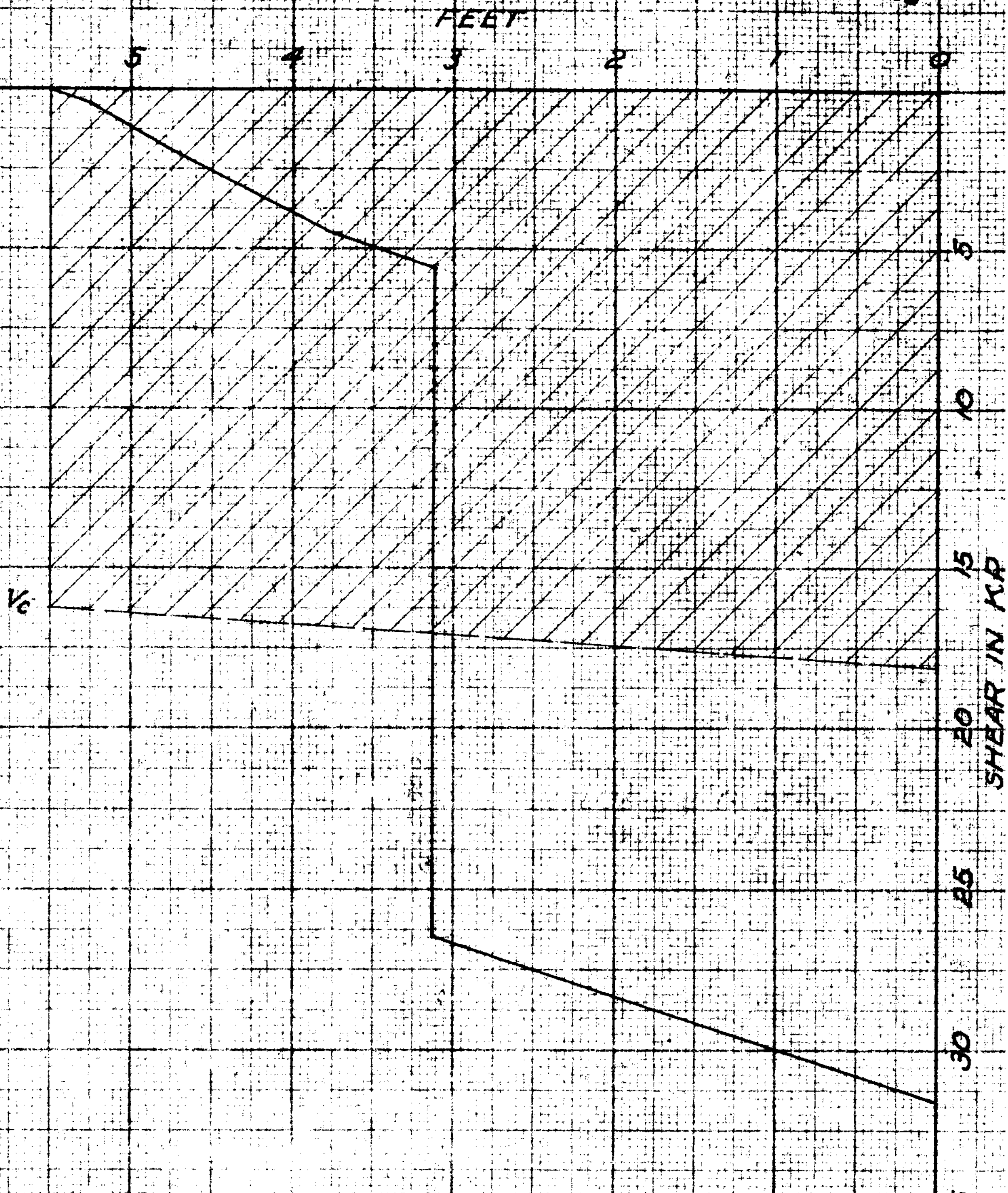


FIG 15

INTERMEDIATE CANTILEVER
DEAD AND LIVE LOAD SHEAR

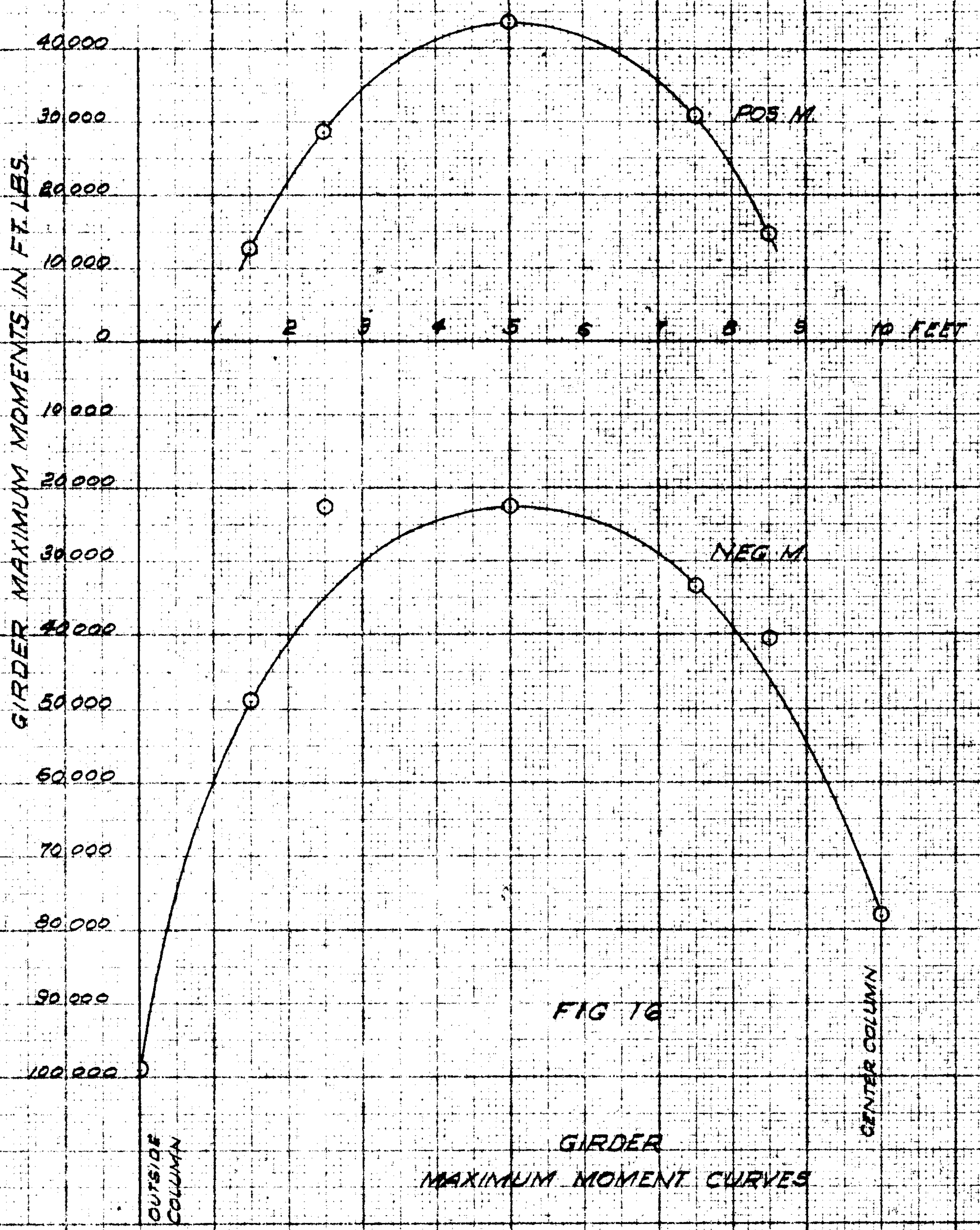


FIG 16

GIRDER
MAXIMUM MOMENT CURVES

STEEL FOR GIRDER

Steel For Cantilever

Only one wheel load can be placed on the cantilever at one time; hence the condition of greatest shear for the cantilever is represented by the combination of dead load and one wheel placed as near the curb as possible. See Fig. 15 for cantilever shear diagram.

For vertical stirrups minimum spacing under the load is .45d or 11.7 inches and at the support is 12.6 inches. Using one-half round stirrups, the spacing required at the support is 11 inches and under the load is 12 inches. Five stirrups at 11 inch spacing are used.

Tension Steel

Dead Load Moment = 33.336 ft lb

Live Load Moment is 20,800 x 3.125

$$= \underline{65,000} \text{ ft lb}$$
$$98,336$$

At Support

$$A_s = \frac{M}{f_s Jd} = \frac{98,336 \times 12}{18,000 \times .86 \times 28} = 2.72 \text{ in}^2$$

Bond At Support

$$\Sigma 0 = \frac{31,591}{125 \times .86 \times 28} = 10.5 \text{ in}^2$$

Stress In Concrete

$$f_c - \frac{2M}{kjbd^2} - \frac{98,336 \times 12 \times 2}{.40 \times .86 \times 15 \times (28)^2} - 584 \ \frac{lb}{in^2}$$

Tension steel required in top of girder at outer column.

$$A_s = \frac{99,182 \times 12}{18,000 \times .86 \times 28} = 2.74 \text{ in}^2$$

Similarly at $2\frac{1}{2}$ ft	A_s = .970 in^2
at 5 ft	A_s = .63 in^2
at $7\frac{1}{2}$ ft	A_s = .920 in^2
at 10 ft	A_s = 2.160 in^2

For the tension steel required at the bottom of the beam we have a tee beam. The allowable width at the top may be half the distance between girders or one quarter the span, using the lesser, but not over six feet. This requires

$$b = \frac{1}{4} \times 10 = 2.5 \text{ feet}$$

also $\frac{b-b'}{2}$ not over 8t

gives $\frac{2.5 - 1.25}{2} = .625 \quad 8 \times 1$

to use diag. 2 we need

$$\frac{M}{f_s bt^2} = \frac{43,336 \times 12}{18,000 \times 30 \times 12 \times 12} = .00669$$

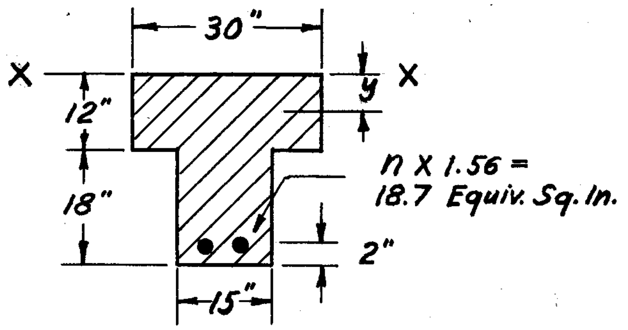
and also

$$\frac{f_c}{f_s} = \frac{1,000}{18,000} = .0555$$

These intersect at a point not on the T beam graph

(Appendix). This suggests a comparatively low moment value for the size of the beam. To compute the I for this beam we must assume the tension steel.

Assuming 2 one-inch round bars for tension steel and neglecting the effect of the compression steel, we have the following:



Taking Area Moments about xx, we have the equation

$$18.7 \times 28 - 30 \times y \times \frac{y}{2} = (18.7 - 30y)\ y$$

$$15y^2 \doteq 18.7y = 523.6$$

$$y^2 - 1.2466y - 34.906 = 0$$

solution gives $y = -5.316$ and -6.563 inches

Taking moments about the centroid for a check, we have

$$18.7 \times 22.634 = 30 \times 5.316 \times \frac{5.316}{2}$$

$$423.25 = 423.9 \text{ which is close enough.}$$

Moment arm of Tensile Force $= 28 - \frac{1}{3} - \times 5.316 = 26.23$

$$f_s = \frac{43.336 \times 12}{3.12 \times 26.23} = 6{,}340 \text{ lb/in}^2$$

Both these stresses are low. In as much as the floor sizes are already fixed, nothing can be done about the concrete stress. The steel stress can be increased by reducing the size of the rods, but since they will be useful also for bond near the supports, they will be used as assumed. For bond requirements, we have at the top, at column,

$$0 = \frac{45,000}{125 \times .86 \times 28} = 14.9 \text{ in}^2$$

at quarter point,

$$0 = \frac{31,000}{125 \times .86 \times 28} = 10.3 \text{ in}^2.$$

For bond requirements at the bottom of the beam, we note from the curve of maximum moments that a positive moment may be produced in the bottom of the beam as close to the support as about one foot. Corresponding to this, the shear will be that due to D. L. plus approximately 25,000 lb from L.L. This gives

$$0 = \frac{34,000}{125 \times .86 \times 28} = 11.3 \text{ in}^2.$$

Compressive stress in concrete over the column supports is

$$f_c = \frac{99,182 \times 12 \times 2}{.40 \times .86 \times 15 \times (28)^2} = 588 \text{ lb/in}^2.$$

For the girders over the center pier, we have the same live loads and dead loads only slightly different but smaller. The same curves of maximum moments will be used for the design of the pier girders.

For positive moment in the center, we have an inverted L beam. If we used as width half of that allowed for the T-Beams, we will have a width of 15 inches which is only the width of the beam itself. From the detail of the floor at the expansion joint it may be seen that the floor extends 3 inches beyond the edge of the girder. If we suppose an equal amount on the other side to be effective, we will have a width at the top of 15 - 6 = 21 inches. From this, we get

$$\frac{M}{f_s bt^2} = \frac{43{,}336 \times 12}{18{,}000 \times 21 \times 12 \times 12} = .00956.$$

As in the case of the other girders this point also falls off the T-Beam graph. We compute the location of the N.S. of this beam also.

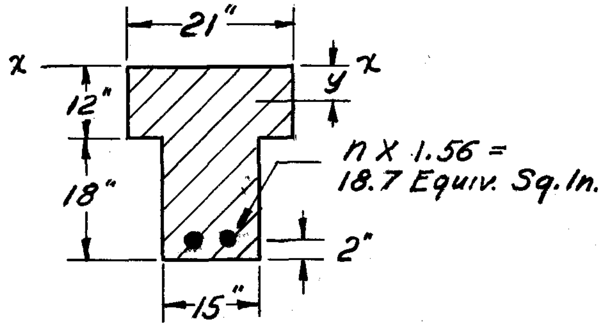


Fig.

Taking Area Moments about xx, we have the equation

$$18.7 \times 28 - 21y \times \frac{y}{2} = (18.7 - 21y)\, y$$

$$10.5y^2 = 18.7y = 523.6$$

$$y^2 - 1.78\,y - 49.87 = 0$$

$$y = -\,8.01 \text{ and } -\,6.23 \text{ inches.}$$

Taking moments about the centroid for a check, we have

$$18.7 \times 21.77 = 6.23 \times 21 \times \frac{6.23}{2}$$

$$407.1 = 407.5 \text{ which is close enough.}$$

Moment arm of the tensile force is

$$28 - \frac{1}{3} \times 6.23 = 25.92 \text{ inches}$$

$$f_s = \frac{43{,}336 \times 12}{3.12 \times 25.92} = 6{,}420.\ \text{lb/in}^2$$

$$f_c = \frac{2 \times 43{,}336 \times 12}{6.23 \times 21 \times 26.23} = 302.\ \text{lb/in}^2$$

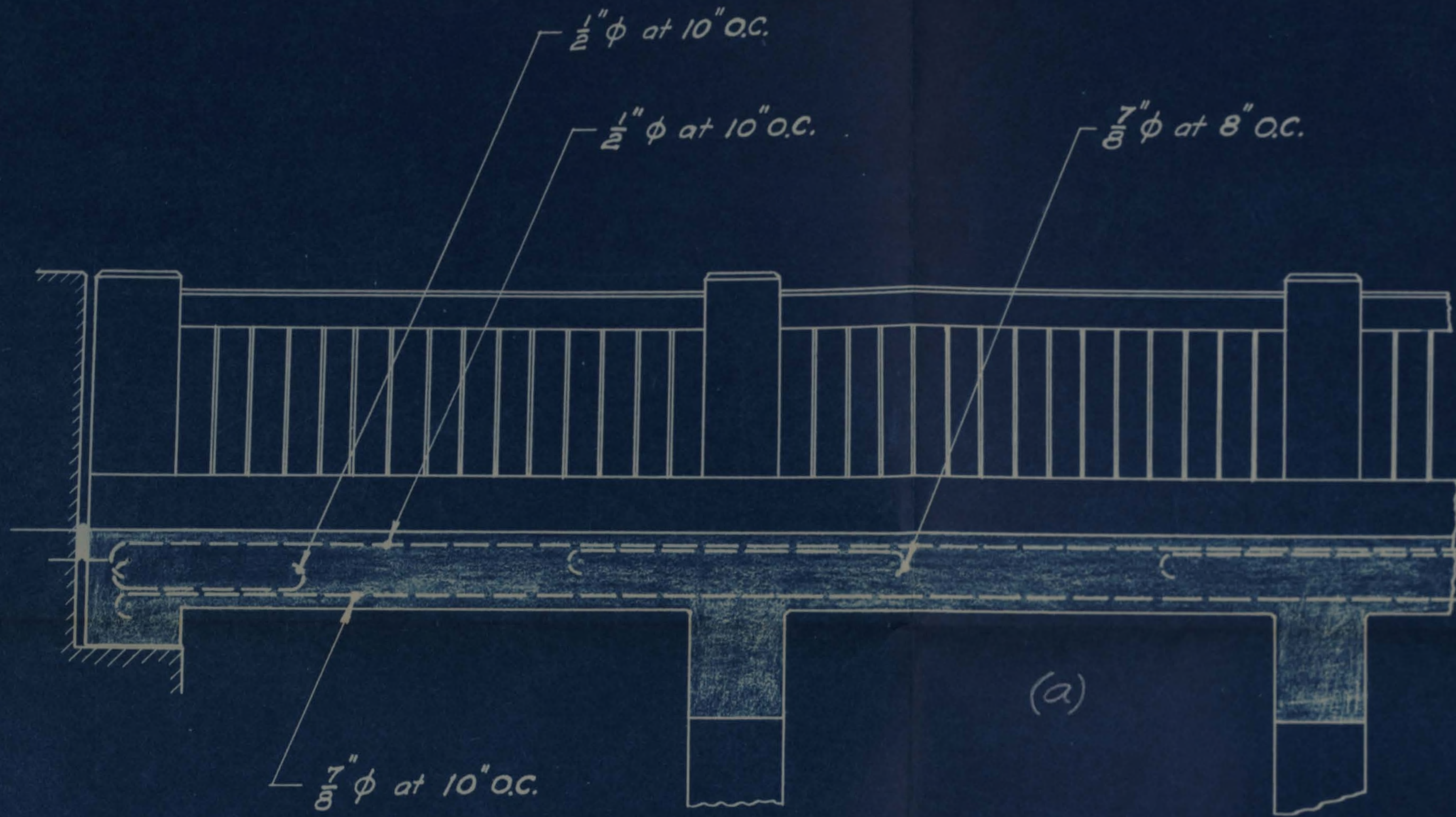
Over the column supports the f_c stress for the pier girder will be no larger than for the typical girders.

The steel selected for the girder is shown on Fig. 17. At the top 2 = 1" round bars are carried through from one end to the other of the girder and hooked at the ends. This provides adequate tension area for all sections between points of inflection on the girder and also provides the best possible anchorage for the cantilever tension steel. The additional tension area and bond surface required over the supports are provided by 3 = $\frac{7}{8}$ round bars which are carried to the quarter points and hooked.

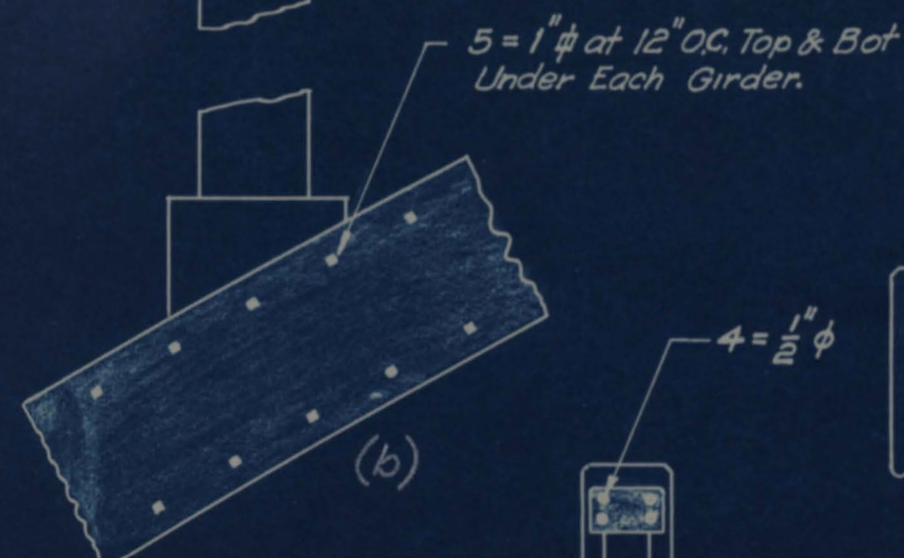
On the bottom 2 = 1" round bars are carried through from outer column to outer column. This provides some more tension area than is required at the center. This, however, provides some margin of safety to allow for the possibility that some discontinuity developing over the supports might require the girder to act more like a simple beam. The extra surface required for bond at the supports is provided for by 2 = $\frac{7}{8}$ round bars extending to the quarter point and hooked.

For stirrups, minimum spacing is 12.6 inches. This is adequate between quarter points and the spacing of $\frac{1}{2}$" round U stirrups there is made 12 inches. From the quarter points toward the supports the spacing is made 8, 7, and 6 inches. For the cantilever, the spacing is made 11 inches to the curb. No stirrups are required beyond the curb.

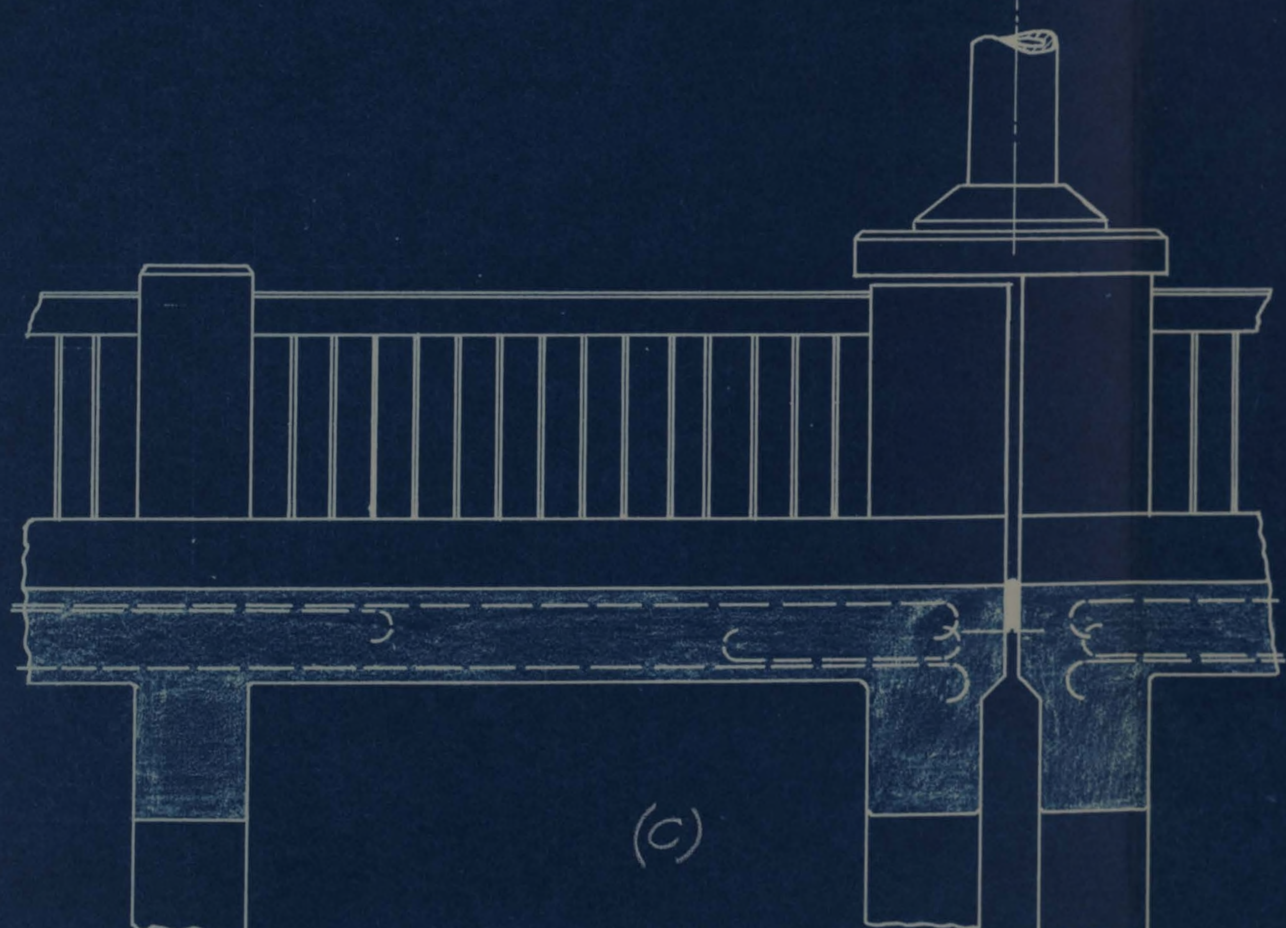
The loads on the pier girders are slightly less, but the difference is so small that the same steel will be used for all girders.



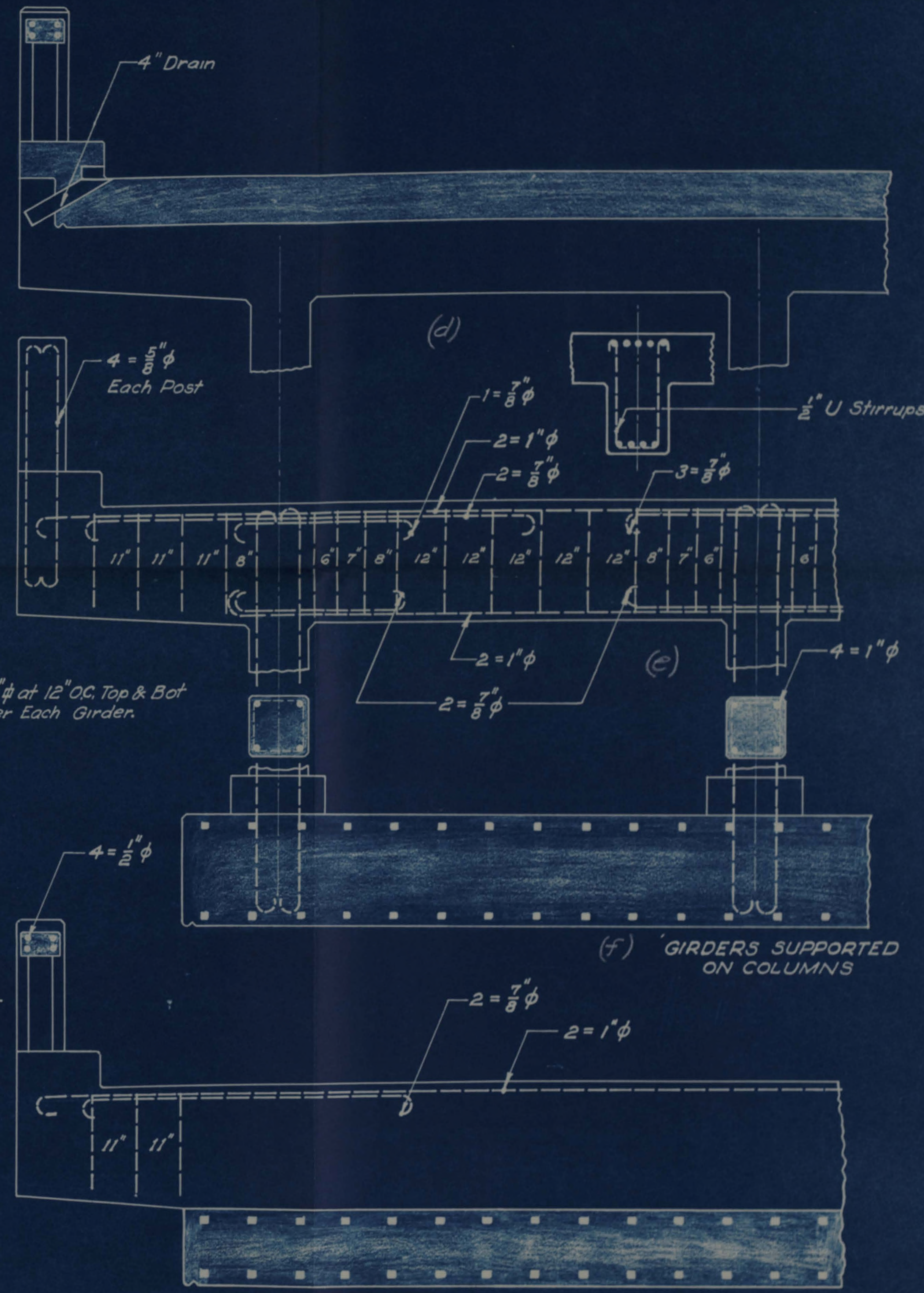
LONGITUDINAL FLOOR SECTION
NEAR ABUTMENT



TRANSVERSE
ARCH REINFORCEMENT
UNDER GIRDERS



LONGITUDINAL FLOOR SECTION
NEAR CENTER PIER



GIRDERS SUPPORTED
ON COLUMNS

GIRDERS SUPPORTED
ON ARCH RIBS

9

DETAILS OF SUPERSTRUCTURE
BRIDGE OVER PEACHTREE CREEK
SCALE 1/2" = 1'-0"

MAY 17 1941

DRAWN BY F.M.H.

CHAPTER V

DESIGN OF ARCH RIB

The bridge has two spans with an elastic pier in the center. As both spans are alike, the dead load stresses will be the same in each. The live load stresses were calculated using two assumptions: first, using the same span as for the dead load calculation and assuming both ends to be rigidly fixed; second, considering the pier as elastic and using as the span twice the distance from the crown of the arch to the centerline of the pier.

The arch axis was laid out to coincide as nearly as practical with the dead load equilibrium polygon. Since this can be drawn only when the shape and thickness of the arch are known, it is necessary to assume these for a preliminary analysis. The preliminary analysis was made using the tables on symmetrical arches prepared by Mr. Whitney.[5] After several preliminary trials the crown was taken as 19 inches thick, and the springing thickness was taken as 28 inches. The thickness was made to increase uniformly with respect to arch axis length measured from the crown.

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[5]Charles S. Whitney, "Design of Symmetrical Concrete Arches," American Society of Civil Engineers No. 88, pp. 931-1103.

Using these thicknesses and the arch axis curve, obtained from Mr. Whitney's tables, which most nearly fitted the given data, a preliminary dead load equilibrium polygon was drawn. From this a revision in the shape of the arch axis was made; another estimate of arch axis weight was made, and another dead load equilibrium polygon drawn. This last equilibrium polygon was found quite close enough to the one which preceded it to indicate that no further revision would appreciably affect the results.

The final result is given here. The equilibrium polygon is shown on Fig. 18. The calculation of the weights used in its construction is indicated here in Table VIII which follows. The portion of arch rib used extended from the crown to the outside of the center pier. The arch rib was divided into lengths four feet long measured horizontally, with the exception of the portion adjacent to the center pier, which was used as seven feet long measured horizontally.

The first portion is called "a"; the weight carried to the arch by the girder nearest the crown is called "b"; the weight of the next portion of arch rib is called "c", etc. All weights are for the entire width of bridge.

TABLE VIII

ARCH DEAD LOAD WEIGHTS

$a = 4 \times \frac{19.4}{12} \times 150$	= 23,250 lb.
$b = 2 \times 10{,}790 - 32{,}800$	= 54,380
$c = 4.03 \times \frac{20.3}{12} \times 24 \times 150$	= 24,600
$d = 4.03 \times \frac{21.3}{12} \times 24 \times 150$	= 25,800
$e = 53{,}380 - \frac{18.34}{12} \times 125 \times 24 \times 150$	= 61,260
$f = 4.375 \times \frac{22.5}{12} \times 24 \times 150$	= 29,500
$g = 4.375 \times \frac{23.5}{12} \times 24 \times 150$	= 30,850
$h = 54{,}380 - 3 \times 450 - 3 \times 965$	= 58,630
$k = 4.86 \times \frac{24.5}{12} \times 24 \times 150$	= 35,800
$l = 4.87 \times \frac{25.5}{12} \times 24 \times 150$	= 37,300
$m = 54{,}380 - 3 \times 600 - 3 \times 2125$	= 62,550
$n = 9.75 \times 2.22 \times 24 \times 150$	= 77,900

To facilitate the construction of the dead load equilibrium polygon, the value of the string at the crown was computed by taking moments about the point formed by the intersection of the assumed axis and the near side of the pier, extended. The sum of the moments of the various dead load weights about this point equals the moment of the crown string about the same point. The rise chosen for this string was 17.6 feet, as the string does not pass precisely through the crown itself. The values of these arch dead load weights and their moments about the assumed point are given in the following table.

TABLE IX

ARCH DEAD LOAD WEIGHT MOMENTS

ABOUT SPRINGING LINE

77,900	x 3.50	=	272,650	lb ft
62,550	x 7.00	=	437,850	
37,300	x 9.00	=	335,700	
35,800	x 13.00	=	465,400	
58,630	x 15.00	=	879,450	
30,850	x 17.00	=	524,450	
29,500	x 21.00	=	619,500	
61,260	x 23.00	=	1,408,980	
25,800	x 25.00	=	645,000	
24,600	x 29.00	=	713,400	
54,380	x 31.00	=	1,685,780	
23,250	x 33.00	=	767,250	
			8,755,410	

This divided by 17.6 gives the value of the crown string as 497,460 lb.

After final construction of the equilibrium polygon, the axis was rectified, the crown and springing thicknesses laid off, and straight lines used to connect them. From this the various thicknesses were obtained for use in a scale layout of the rib. This is shown in Fig. 18.

The single span analysis is presented first. The method of analysis used is that developed by the Bureau of

Public Roads.[6] The method is also used by the Civil Engineering Department of the Georgia School of Technology. The tabulation used here is similar to that presented by Dr. F. C. Snow, Head of the Civil Engineering Department at the Georgia School of Technology, in his "Concrete Notes".[7] It is assumed that the reader is familiar with the theory of the right symmetrical arch, and no further mention will be made here of the detailed steps used in completing the tabulation.

As the equation of the arch axis cannot be conveniently obtained, the necessary integration is performed arithmetically. The number of segments used for the entire arch is twenty. These were made by dividing the span into twenty equal horizontal divisions and projecting upward to the arch axis. The lengths along the arch axis so obtained were called ds. The thickness of the arch rib at the center of a division is called h; its distance above a horizontal line drawn through the center of the skewback is called y, and was obtained by scaling a large drawing of the rib.

The longitudinal reinforcement consists of 1¼ inch square bars at the crown spaced 12 inches o.c. both top and

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[6] W. P. Linton and C. D. Geisler, "Analysis of Concrete Arches," reprint from Public Roads, Vol. 8, Nos. 4 & 5, U. S. Department of Agriculture Bureau of Public Roads. pp. 24.

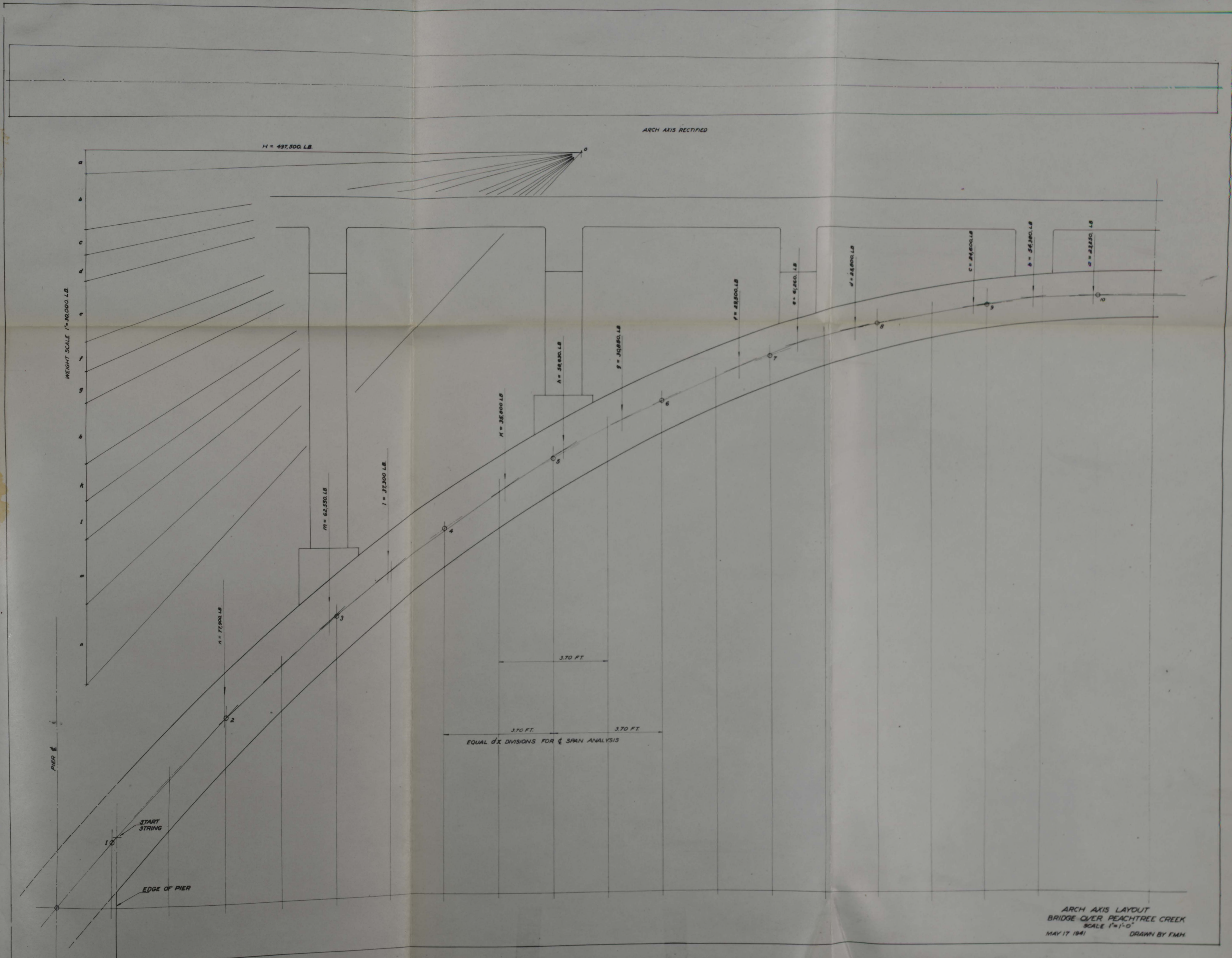
[7] Op.Cit.

bottom, and at the springing it consists of 1 inch square bars spaced 5 inches o.c. both top and bottom. The transition in the reinforcement takes place near the center of the arch ring between the long column and short column bents.

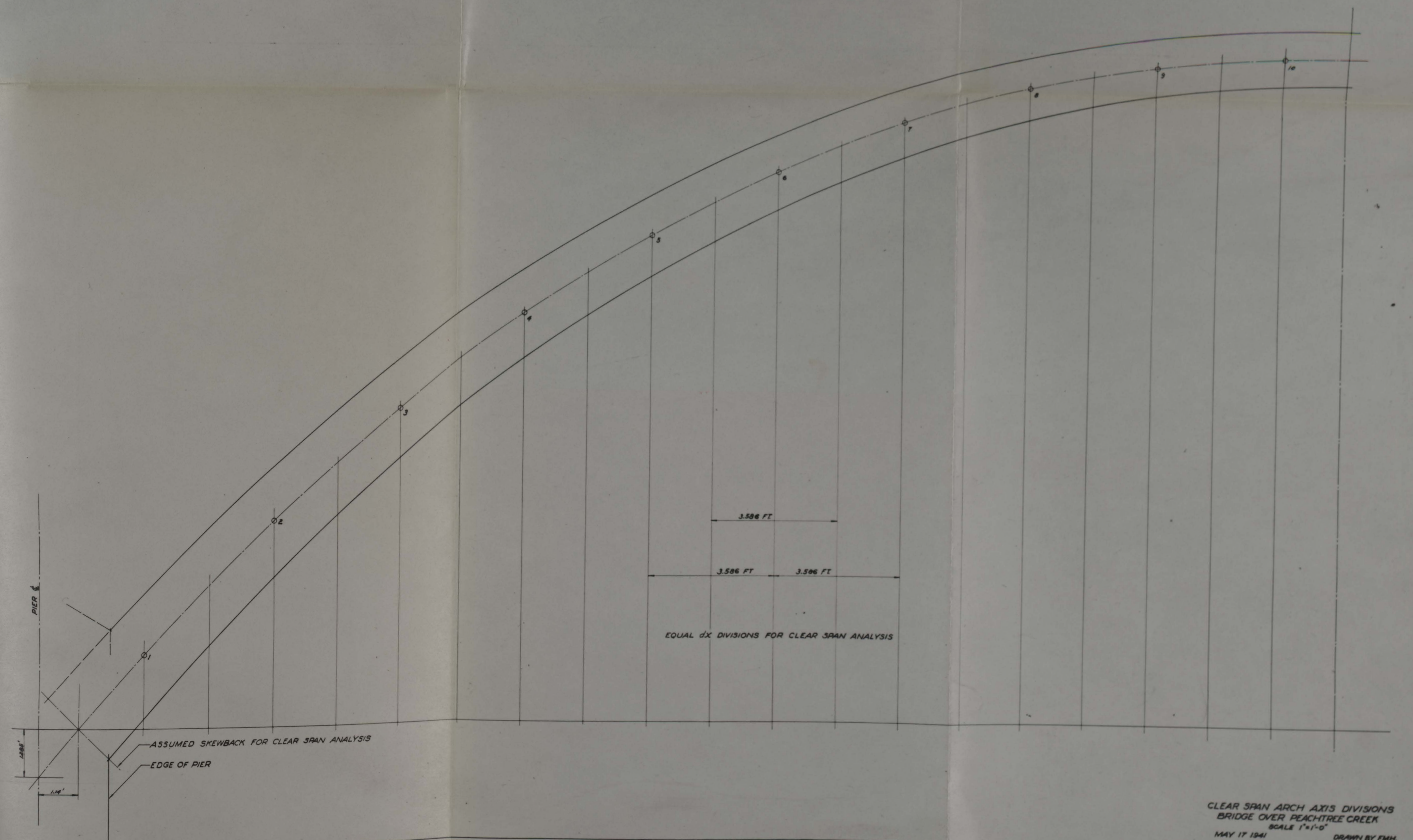
All the live load and part of the dead load is applied to the arch through columns resting on the arch and supporting the floor girders. The tabulation analysis for the arch rib gives M_0 H_0 and V_0 only for the points used in the initial subdivision of the rib, and these do not coincide with the points of application of the column loads. To obtain M_0 H_0 and V_0 for loads applied at the columns, it was necessary to draw influence lines for these functions. From these influence lines, which are reproduced on one of the following sheets, the desired values were obtained. With M_0 H_0 and V_0 known, it was possible to calculate, by the principles of statics, influence line ordinates for the moment at any other point on the arch rib. Influence lines were constructed for M_{G1} M_{G2} M_{G3} and M_{G4}.

Rib shortening is taken care of by the method of solution used in the arch-rib analysis. Temperature stresses were calculated using a rise of 30° F. and a drop of 40° F.

The computation of final maximum stresses is shown in Table XXIII. The live load per foot of arch width is computed with all three lanes loaded. It is 124,800 lb distributed over 24 feet and equals 5,200 lb per foot of arch width. It may be seen from this table that on the basis of the assumption made the greatest fiber stress in the concrete is about 557 lb per square inch and is found in the rib at the springing. The maximum stress at the crown girder is nearly as great. These two points on the arch rib are therefore the points of greatest fiber stress.



ARCH AXIS LAYOUT
BRIDGE OVER PEACHTREE CREEK
SCALE 1" = 1'-0"
MAY 17 1941 DRAWN BY F.M.H.



CLEAR SPAN ARCH AXIS DIVISIONS
BRIDGE OVER PEACHTREE CREEK
SCALE 1" = 1'-0"
MAY 17 1941 DRAWN BY F.M.H.

TABLE X COMPUTATION OF ELASTIC WEIGHTS

1	2	3	4	5	6	7	8	9	10	11
POINTS	$h = A$	h^3	$I_c = \frac{h^3}{12}$	$\frac{h}{2}$	$\frac{h}{2} - d'$	$\left[\frac{h}{2} - d'\right]^2$	$I_s = \left[\frac{h}{2} - d'\right]^2 nA_s$	$I = I_c + I_s$	ds	$\frac{ds}{I} = \Delta$
0	2.33									
1	2.31	12.34	1.029	1.155	.985	.970	.296	1.325	5.26	3.97
2	2.21	10.80	.901	1.105	.935	.875	.267	1.168	4.86	4.16
3	2.125	9.60	.80	1.062	.892	.795	.243	1.043	4.56	4.37
4	2.043	8.54	.712	1.021	.851	.724	.221	.933	4.29	4.60
5	1.967	7.62	.635	.983	.813	.661	.158	.793	4.08	5.15
6	1.892	6.77	.565	.946	.776	.602	.144	.709	3.93	5.55
7	1.820	6.03	.502	.910	.740	.548	.131	.633	3.78	5.97
8	1.750	5.36	.447	.875	.705	.497	.119	.566	3.68	6.50
9	1.670	4.66	.388	.835	.665	.442	.105	.493	3.62	7.34
10	1.617	4.22	.352	.808	.638	.407	.097	.449	3.59	8.00
									$\frac{1}{2}\Sigma =$	55.61

SPAN = 71.72 FT

RISE = 18.55 FT.

TABLE XI COMPUTATIONS OF V_0

—	12	13	14	15	16	17	18
POINTS	$Z-20$ WHERE $Z=\frac{2x}{d\bar{x}}$	$Q=(Z-20)\Delta$	$-\sum_a^{1}(Z-20)\Delta$	$-\frac{1}{2}\sum\left(Z-\frac{2a}{d\bar{x}}\right)Q$	$Z^2+(40-Z)^2$	$\left[Z^2+(40-Z)^2\right]\Delta$	V_0
0							
1	-19	-75.43	75.43	5997.85	1522	6042.34	1.00
2	-17	-70.72	146.15	5922.42	1378	5732.48	.98472
3	-15	-65.55	211.70	5776.27	1250	5462.50	.96305
4	-13	-5980	271.50	5564.57	1138	5234.80	.92776
5	-11	-56.65	328.15	5293.07	1042	5366.30	.88219
6	-9	-49.95	378.10	4964.92	962	5339.10	82778
7	-7	-41.79	419.89	4586.82	898	5361.06	76474
8	-5	-32.50	452.39	4166.93	850	5525.00	.69473
9	-3	-22.02	474.41	3714.54	818	6004.12	.61931
10	-1	- 8.00	482.41	3240.13	802	6416.00	.54021
		-482.41	3240.13			56483.70	

$$V_0 = \frac{-\frac{1}{2}\sum_a^{7}\left(Z-\frac{2a}{d\bar{x}}\right)(Z-20)\Delta}{F}$$

SPAN = 71.72
RISE = 18.55

TABLE XII COMPUTATIONS OF H_o

19	20	21	22	23	24	25	26	27	28	29	30
POINTS	Y	Δ	$Y\Delta$	$Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}$	$R=\Delta\left[Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right]$	$\sum_0^1\Delta\left(Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right)$	$\frac{1}{2}\sum_a^1\left(Z-\frac{2a}{dx}\right)R$	$Y\Delta\left(Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right)$	$\cos\phi$	$\frac{\cos\phi}{A}$	H_o
0	0					+ .005					
1	1.99	3.97	7.900	-11.7876	-46.797	46.802	0	-93.126	.681	.29481	0
2	5.56	4.16	23.129	-8.2176	-34.185	80.987	-46.802	-190.068	.736	.33303	.05731
3	8.59	4.37	37.538	-5.1876	-22.670	103.657	-127.789	-194.735	.784	.36894	.15648
4	11.18	4.60	51.428	-2.5976	-11.949	115.606	-231.446	-133.590	.835	.40871	.28341
5	13.33	5.15	68.649	-0.4476	-2.305	117.911	-347.052	-30.725	.878	.44637	.42497
6	15.10	5.55	83.805	1.3224	7.339	110.572	-464.963	110.819	.915	.48362	.56935
7	16.48	5.97	98.385	2.7024	16.133	94.439	-575.535	265.872	.948	.52088	.70475
8	17.51	6.50	113.815	3.7324	24.260	70.179	-669.974	424.792	.974	.55657	.82039
9	18.17	7.34	133.368	4.3924	32.240	37.939	-740.153	585.801	.990	.59281	.90633
10	18.52	8.00	148.160	4.7424	37.939	0	-778.092	702.630	.999	.61781	.95279
	18.55				+ .005	778.092		1447.670		4.62355	4.87578

$$H_o=\frac{\frac{1}{2}\sum_a^1\left(Z-\frac{2a}{dx}\right)\left(Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right)\Delta}{C}$$

$$F=\frac{1}{2}\sum Z^2\Delta-200\Sigma\Delta=5997.85$$

$$Z=\frac{2X}{dx}$$

$$\frac{\Sigma Y\Delta}{\Sigma\Delta}=13.7776$$

$$dx=3.586$$

$$C=\frac{1}{dx}\sum_a^1 Y\Delta\left(Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right)+\sum\frac{\cos\phi}{A}=816.648$$

$$H_t=\frac{20e+E}{C}=52.9t$$

SPAN = 71.72

RISE = 18.55

TABLE XIII COMPUTATIONS FOR M_o

31	32	33	34	35	36	37	38	39	40	41
POINT	Z	H_o	V_o	Δ	$\sum_a^1 \Delta$	$\frac{1}{2}\sum_a^1 (Z-\frac{2a}{dx})\Delta$	$\frac{dx}{\Sigma\Delta}$ (Col 37)	$\frac{\Sigma Y\Delta}{\Sigma\Delta} H_o$	$-20\frac{dx}{2}V_o$ -35.86	M_o
0	0									
1	1	0	1.00	3.97	107.25	1056.59	34.0666	0	-35.8600	-1.7934
2	3	.05731	.98472	4.16	103.09	949.34	30.6086	0.78959	-35.3121	-3.9139
3	5	.15648	.96305	4.37	98.72	846.25	27.2848	2.15592	-34.5350	-5.0943
4	7	.28341	.92776	4.60	94.12	747.53	24.1018	3.90471	-33.2695	-5.2630
5	9	.42497	.88249	5.15	88.97	653.41	21.0672	5.85507	-31.6461	-4.7238
6	11	.56935	.82778	5.55	83.42	564.44	18.1987	7.84427	-29.6842	-3.6412
7	13	.70475	.76474	5.97	77.45	481.02	15.5090	9.70976	-27.4236	-2.2048
8	15	.82039	.69473	6.50	70.95	403.57	13.0119	11.30301	-24.9130	-0.5981
9	17	.90633	.61931	7.34	63.61	332.62	10.7243	12.48705	-22.2085	+1.0029
10	19	.95279	.54021	8.00	55.61	269.01	8.6734	13.12716	-19.3719	2.4287
10'	19	.95279	.45979	8.00	47.61	213.40	6.8804	13.12716	-16.4881	3.5195
9'	17	.90633	.38069	7.34	40.27	165.79	5.3454	12.48705	-13.6515	4.1809
8'	15	.82039	.30527	6.50	33.77	125.52	4.0470	11.30301	-10.9470	4.4030
7'	13	.70475	.23526	5.97	27.80	91.75	2.9582	9.70976	-8.4364	4.2315
6'	11	.56935	.17222	5.55	22.25	63.95	2.0619	7.84427	-6.1758	3.7304
5'	9	.42497	.11751	5.15	17.10	41.70	1.3445	5.85507	-4.2139	2.9857
4'	7	.28341	.07224	4.60	12.50	24.60	0.7932	3.90471	-2.5905	2.1074
3'	5	.15648	.03695	4.37	8.13	12.10	0.3901	2.15592	-1.3250	1.2210
2'	3	.05731	.01528	4.16	3.97	3.97	0.1280	0.78959	-0.5479	0.3697
1'	1	0	0	3.97	0	0	0	0	0	0
0'	0									
		9.75156	10.000	111.22	1056.59	7046.56	227.1950	134.35308	-358.600	2.9482

$$M_o = \frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^1\left(Z-\frac{2a}{dx}\right)\Delta + H_o\frac{\Sigma Y\Delta}{\Sigma\Delta} - 20\frac{dx}{2}V_o$$

$$\frac{dx}{\Sigma\Delta} = .032242 \qquad -20\frac{dx}{2} = -35.86$$

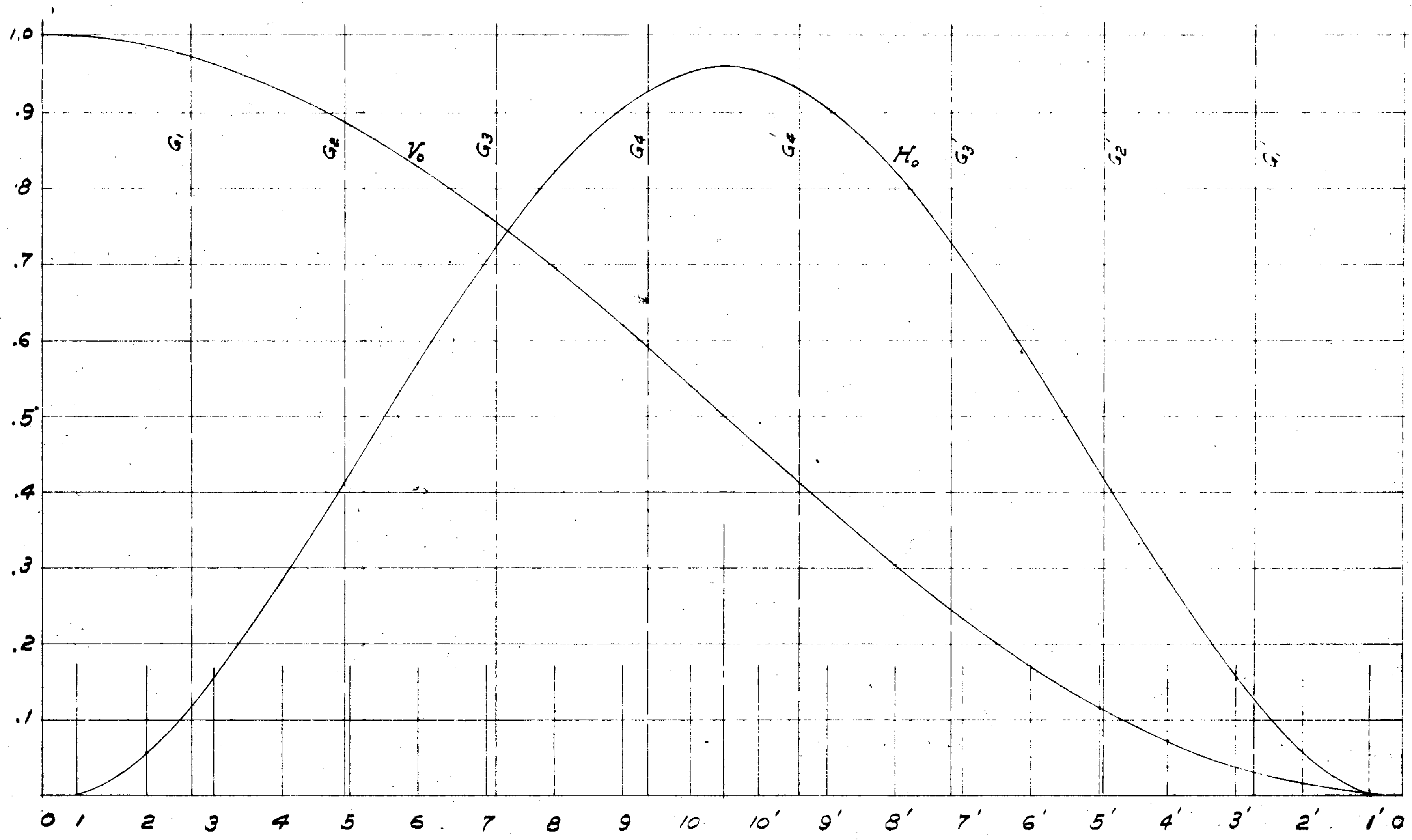
$$\frac{\frac{1}{2}\text{ Sum } 22}{\frac{1}{2}\text{ Sum } 11} \times \text{Sum } 33 = \text{Sum } 39$$

$$-20\frac{dx}{2}(\text{Sum } 34) = \text{Sum } 40$$

Sum 38 + Sum 39 + Sum 40 = Sum 41

SPAN = 71.72

RISE = 18.55



INFLUENCE ORDINATES FOR V_0 AND H_0
CLEAR SPAN ANALYSIS
FIG 20

TABLE XIV COMPUTATION M_{G_1} TABLE XV COMPUTATION M_{G_2}

42	43	44	45	46	47	48	49	50	51	52	53
PT	H_o	V_o	M_o	$V_o m$	$x-m$	$-H_o y$	M_{G_1}	$V_o m$	$x-m$	$-H_o y$	M_{G_2}
0	.00	1.000	.000	7.860	-7.86	.0	.00	15.860	-15.860	.00	.00
1	.00	1.000	-1.7934	7.860	-6.067	.0	.00	15.860	-14.067	.00	-.0004
2	.05731	.98472	-3.9139	7.740	-2.481	-.4413	.9038	15.6176	-10.481	- .7565	.4653
G_1	.11900	.97300	-4.7900	7.6478		- .9163	1.9415	15.4317	8.000	-1.5708	1.0709
3	.15648	.96305	-5.0943	7.5696		-1.2049	1.2704	15.274	-6.895	-2.0655	1.2192
4	.28341	.92776	-5.2630	7.2922		-2.1822	- .1530	14.7143	-3.309	-3.7410	2.4013
G_2	.41200	.88800	-4.7900	6.9797		-3.1720	- .9823	14.084		-5.4384	3.8556
5	.42497	.88249	-4.7238	6.9365		-3.2722	-1.0595	13.9963		-5.6096	3.6626
6	.56935	.82778	-3.6412	6.5063		-4.3840	-1.5189	13.129		-7.5154	1.9724
7	.70475	.76474	-2.2048	6.0108		-5.4265	-1.6205	12.1287		-9.3027	.6212
8	.82039	.69473	- .5981	5.4605		-6.3170	-1.4546	11.0184		-10.8291	- .4088
9	.90633	.61931	1.0029	4.8677		-6.9787	-1.1081	9.8222		-11.9635	-1.1384
10	.95279	.54021	2.4287	4.2460		-7.3365	- .6618	8.5680		-12.5768	-1.5801
10′	.95279	.45979	3.5195	3.6139		-7.3365	- .2031	7.2922		-12.5768	-1.7651
9′	.90633	.38069	4.1809	2.9922		-6.9787	.1944	6.0377		-11.9635	-1.745
8′	.82039	.30527	4.4030	2.3994		-6.3170	.4854	4.8416		-10.8291	-1.5845
7′	.70475	.23526	4.2315	1.8491		-5.4265	.6541	3.7312		-9.3027	-1.340
6′	.56935	.17222	3.7304	1.3536		-4.3840	.700	2.7314		-7.5154	-1.0536
5′	.42497	.11751	2.9857	.9236		-3.2722	.6371	1.8637		-5.6096	- .7602
4′	.28341	.07224	2.1074	.5678		-2.1822	.4930	1.1457		-3.7410	- .4879
3′	.15648	.03695	1.2210	.2904		-1.2049	.3065	.5860		-2.0655	- .2585
2′	.05731	.01528	.3697	.1201		- .4413	.0485	.2423		- .7565	- .1445
1′	.00	.00	.00	.00		.00	.00	.00		.00	.00
0′	.00	.00	.00	.00		.00	.00	.00		.00	.00
	10.28256	12.8610	-6.6318	101.0872	-16.408	-79.1749	-1.1271	203.9754	-58.612	-135.7294	3.0015

y_{G_1} = 7.70 FT. y_{G_2} = 13.20 FT.

m_{G_1} = 7.86 FT. m_{G_2} = 15.86 FT.

SPAN = 71.72

RISE = 18.55

TABLE XVI COMPUTATION M_{G_3} TABLE XVII COMPUTATION M_{G_4}

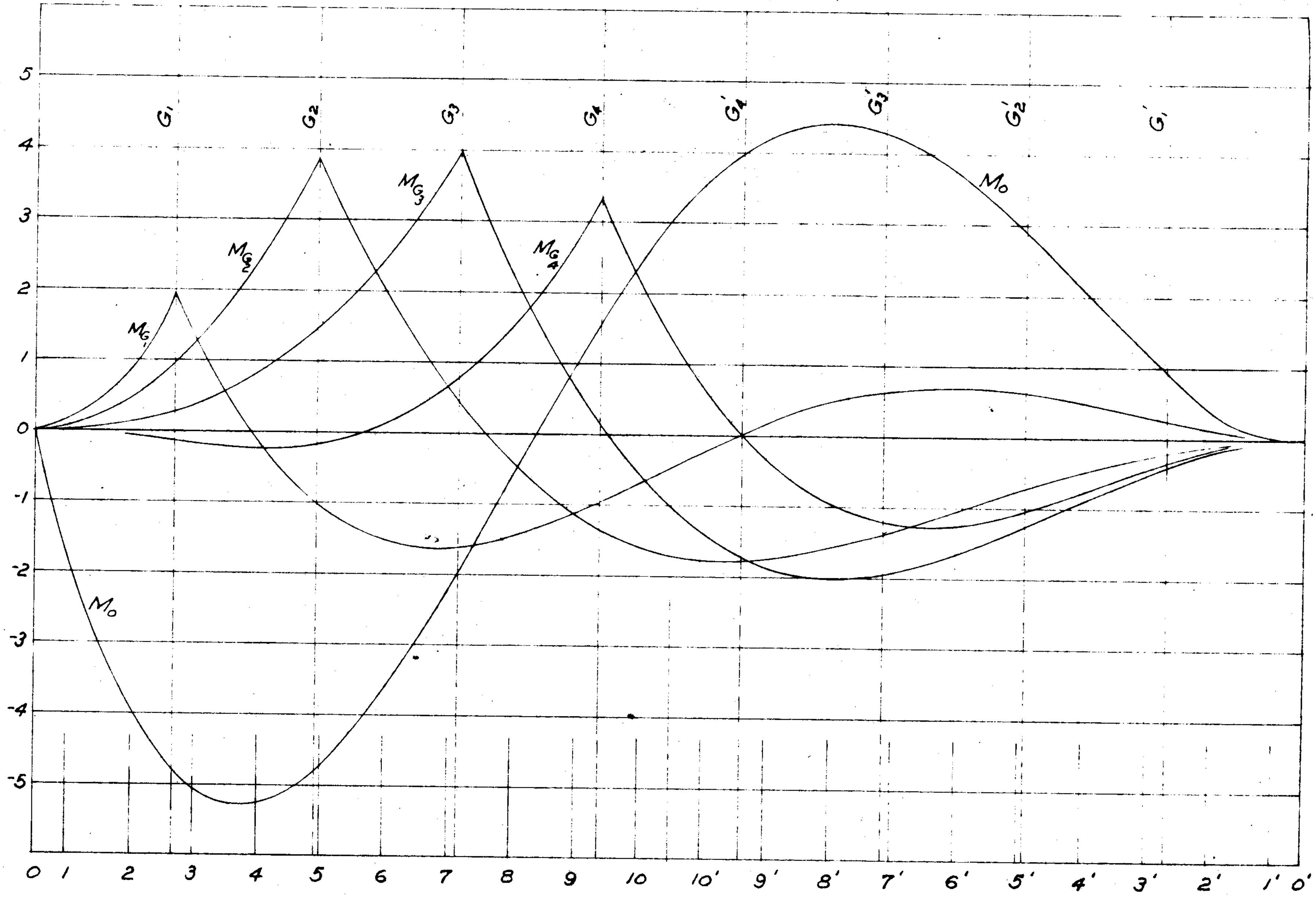
54	55	56	57	58	59	60	61	62	63	64	65
PT	H_o	V_o	M_o	$V_o m$	$x-m$	$-H_o y$	M_{G_3}	$V_o m$	$x-m$	$-H_o y$	M_{G_4}
0	.00	1.000	.00	23.860	-23.860	- .00	.00	31.860	-31.860	.00	- .000
1	.00	1.000	-1.7934	23.860	-22.067	- .00	- .0004	31.860	-30.067	.00	- .0004
2	.05731	.98472	-3.9139	23.4954	-18.481	- .9562	.1443	31.373	-26.481	-1.05221	- .07411
3	.15648	.96305	-5.0943	22.9784	-14.895	-2.6108	.3780	30.683	-22.895	-2.87300	- .17930
4	.28341	.92776	-5.2630	22.1363	-11.309	-4.7287	.8350	29.558	-19.309	-5.20340	- .21740
5	.42497	.88249	-4.7238	21.0562	-7.723	-7.0906	1.5180	28.1161	-15.723	-7.80245	- .13315
6	.56935	.82778	-3.6412	19.7508	-4.137	-9.4996	2.4730	26.373	-12.137	-10.45326	.14154
7	.70475	.76474	-2.2048	18.2467	- .551	-11.7587	3.7320	24.3646	- 8.551	-12.93921	.6696
G_3	.72400	.75500	-1.9700	18.0143		-12.0800	3.9640	24.054	- 8.000	-13.29264	.79136
8	.82039	.69473	- .5981	16.5762		-13.6882	2.2900	22.134	- 4.965	-15.06236	1.5085
9	.90633	.61931	1.0029	14.7767		-15.1221	.6580	19.731	- 1.379	-16.64022	2.7147
G_4	.92800	.59000	1.5600	14.0774		-15.4837	.1534	18.797		-17.03810	3.3189
10	.95279	.54021	2.4287	12.8894		-15.8973	- .5796	17.211		-17.49322	2.1465
10'	.95279	.45979	3.5195	10.9706		-15.8973	-1.408	14.649		-17.49322	.67528
9'	.90633	.38069	4.1809	9.0832		-15.1221	-1.858	12.129		-16.64022	- .33032
8'	.82039	.30527	4.4030	7.2837		-13.6882	-2.0015	9.726		-15.06236	- .9333
7'	.70475	.23526	4.2315	5.6133		-11.7587	-1.914	7.4954		-12.93921	-1.2123
6'	.56935	.17222	3.7304	4.1091		-9.4996	-1.660	5.487		-10.45326	-1.23586
5'	.42497	.11751	2.9857	2.8038		-7.0906	-1.3011	3.7438		-7.80245	-1.07295
4'	.28341	.07224	2.1074	1.7236		-4.7287	- .898	2.3015		-5.20340	- .79450
3'	.15648	.03695	1.2210	.8816		-2.6108	- .508	1.1772		-2.87300	- .47480
2'	.05731	.01528	.3697	.3646		- .9562	- .222	.4868		-1.05221	- .19571
1'	.00	.00	.00	.00		.00	.00	.00		.00	.00
0'	.00	.00	.00	.00		.00	.00	.00		.00	.00
	11.40356	12.345	2.5382	294.551	-103.023	-190.2681	3.7950	393.3104	-181.367	-209.3694	+5.11228

y_{G_3} = 16.685 FT. y_{G_4} = 18.36 FT.

m_{G_3} = 23.86 FT. m_{G_4} = 31.86 FT.

SPAN = 71.72

RISE = 18.55



INFLUENCE ORDINATES FOR MOMENTS

CLEAR SPAN ANALYSIS

FIG 21

TABLE XVIII D.L. H_o

66	67	68	69
POINT	DEAD LOAD	UNIT H_o	H_o
0			
1	1822.5	0	0
2	1611.	.05731	92.3
G_1	2606.	.11800	307.5
3	1453.5	.15648	227.4
4	1314.6	.28341	372.6
G_2	2443.	.4140	1011.4
5	1203.8	.42497	511.6
6	1115.3	.56935	635.0
7	1031.9	.70475	727.2
G_3	2553.	.7230	1845.8
8	966.	.82039	792.5
9	906.75	.90633	821.8
G_4	2266.	.9270	2100.6
10	870.75	.95279	829.6
	22,164.		10,275.3
		2x =	20,550.6

TABLE XIX D.L. V_G

70	71	72	73
PT	V_o	$\Sigma_o^b \omega \cdot ds$	V_p
0	22,164.	0	22,164.
G_1	22,164	3,725.	18,439.
G_2	22,164	9,320.	12,844.
G_3	22,164	14,215.	7,949.
G_4	22,164	18,925.	3,239.

TABLE XX TEMPERATURE MOMENTS

POINT	COND.	H_t	$Y - \frac{\Sigma Y\Delta}{\Sigma\Delta}$	M_t
0	+30	1587.	-13.7776	+21,865.
	-40	-2116.	-13.7776	-29,153.
G_1	+30	1587.	-6.0776	+9645.
	-40	-2116.	-6.0776	-12,860.
G_2	+30	1587.	-.5776	+916.
	-40	-2116.	-.5776	-1222.
G_3	+30	1587.	2.9074	-4614.
	-40	-2116.	2.9074	+6152.
G_4	+30	1587.	4.5824	-7272.
	-40	-2116.	4.5824	+9696.

$$M_t = -H_t\left(Y - \frac{\Sigma Y\Delta}{\Sigma\Delta}\right)$$

SPAN = 71.72 FT
RISE = 18.55 FT

TABLE XXI DEAD LOAD MOMENT VALUES

74	75	76	77	78	79	80	81	82
POINT	ds	h	h.ds	h·ds x150	UNIT M_o	M_o	UNIT M_{G_1}	M_{G_1}
0					.00	0.0		
1	5.26	2.31	12.150	1822.5	-1.7934	-3268.4	.00	0.0
2	4.86	2.21	10.740	1611.	-3.9139	-6305.3	.9038	1456.0
G_1				2606.	-4.7900	-12,482.7	1.9415	5059.5
3	4.56	2.125	9.69	1453.5	-5.0943	-7404.6	1.2704	1846.5
4	4.29	2.043	8.764	1314.6	-5.2630	-6918.7	- .1530	-201.1
G_2				2443.	-4.7900	-11702.	- .9823	-2399.7
5	4.08	1.967	8.0253	1203.8	-4.7238	-5686.5	-1.0595	-1275.4
6	3.92	1.892	7.4355	1115.3	-3.6412	-4061.	-1.5189	-1694.
7	3.78	1.820	6.8796	1031.9	-2.2048	-2275.1	-1.6205	-1672.2
G_3				2553.	-1.9700	-5029.4	-1.610	-4110.3
8	3.68	1.750	6.440	966.	- .5981	- 577.7	-1.4546	-1405.1
9	3.62	1.670	6.045	906.75	1.0029	909.4	-1.1081	-1004.7
G_4				2266.	1.5600	3535.	- .960	-2175.3
10	3.59	1.617	5.805	870.75	2.4287	2114.8	- .6618	-576.2
10'	3.59	1.617	5.805	870.75	3.5195	3064.6	- .2031	-176.8
G_4'				2266.	4.00	9064.	.00	0.0
9'	3.62	1.670	6.045	906.75	4.1809	3791.	.1944	176.2
8'	3.68	1.750	6.440	966.	4.4030	4253.3	.4854	4689.
G_3'				2553.	4.290	10,952.4	.620	1583.
7'	3.78	1.820	6.8796	1031.9	4.2315	4366.5	.6541	674.9
6'	3.92	1.892	7.4355	1115.3	3.7304	4160.5	.700	780.7
5'	4.08	1.967	8.0253	1203.8	2.9857	3594.2	.6371	767.
G_2'				2443.	2.900	7084.7	.630	1539.1
4'	4.29	2.043	8.764	1314.6	2.1074	2770.4	.493	648
3'	4.56	2.125	9.69	1453.5	1.2210	1774.7	.3065	445.6
G_1'				2606.	.930	2423.6	.250	651.5
2'	4.86	2.21	10.74	1611.	.3697	595.6	.0485	78.
1'	5.26	2.31	12.150	1822.5	.00	00.	.00	0.
0'					.00			
						-1256.7		+3704.1

SPAN = 71.72 FT.

RISE = 18.55 FT.

TABLE XXI CONTINUED D.L. MOM VALUES

83	84	85	86	87	88	89	90
POINT	h·ds x150	UNIT M_{G_2}	M_{G_2}	UNIT M_{G_3}	M_{G_3}	UNIT M_{G_4}	M_{G_4}
0							
1	1822.5	- .0004	- .73	.00		.0	
2	1611.	.4653	749.6	.1443	232.5	- .07411	-119.4
G_1	2606.	1.0709	2790.7	.280	729.7	- .140	-364.8
3	1453.5	1.2192	1772.1	.3780	549.1	- .1793	-260.6
4	1314.6	2.4013	3156.7	.8350	1097.7	- .2174	-285.8
G_2	2443.	3.8556	9419.2	1.470	3591.2	- .180	-439.7
5	1203.8	3.6626	4409.0	1.5180	1827.3	- .13315	-160.3
6	1115.3	1.9724	2199.8	2.473	2758.1	.14154	157.8
7	1031.9	.6212	641.0	3.7320	3851.0	.6696	690.9
G_3	2553.	.430	1097.8	3.9640	10,120.1	.79136	2020.3
8	966.	- .4088	-394.9	2.290	2212.1	1.5085	1457.2
9	906.75	- 1.1384	-1032.2	.658	596.6	2.7147	2461.5
G_4	2266.	- 1.330	-3013.8	.1534	347.6	3.3189	7520.6
10	870.75	- 1.5801	-1375.9	- .5796	- 504.7	2.1465	1869.0
10′	870.75	- 1.7651	-1536.9	- 1.408	-1226.0	.67528	588.0
G_4'	2266.	- 1.760	-3988.1	- 1.720	-3897.5	.00	0
9′	906.75	- 1.745	-1582.3	- 1.858	-1684.7	- .33032	- 299.5
8′	966.	- 1.5845	-1530.6	- 2.0015	-1933.4	- .9333	- 901.5
G_3'	2553.	- 1.380	-3523.1	- 1.950	-4978.3	- 1.200	-3063.6
7′	1031.9	- 1.340	-1382.7	- 1.914	-1975.0	- 1.2123	-1251.0
6′	1115.3	- 1.0536	-1175.1	- 1.660	-1851.4	- 1.23586	-1378.3
5′	1203.8	- .7602	- 915.1	- 1.3011	-1566.2	- 1.07295	-1291.6
G_2'	2443.	- .700	-1710.1	- 1.250	-3053.7	- 1.040	-2540.7
4′	1314.6	- .4879	- 641.4	- .898	-1180.5	- .7945	-1044.5
3′	1453.5	- .2585	- 375.7	- .508	- 738.4	- .4748	- 690.1
G_1'	2606.	- .2500	- 521.2	- .400	-1042.4	- .3500	- 912.1
2′	1611.	- .1445	- 232.8	- .222	- 357.6	.19571	- 315.3
1′	1822.5	.00	0	.0	0	.00	0
0′							
			+1303.4		+1923.5	+2.20268	+1445.5

TABLE XXII MAX MOMENTS AND THRUSTS-CLEAR SPAN

91	92	93	94	95	96	97	98
POINT	LOAD	M	H	HCOS φ	V	VSIN φ	N
O	D.L.	- 1257	+20550	+ 13686	+ 22164	+ 16534	+30220
	+ C.L.L	+ 24195	+ 4139	+ 2757	+ 1336	+ 997	+ 3754
SIN φ = .746	- C.L.L.	- 28660	+ 1453	+ 968	+ 6087	+ 4541	+ 5509
COS φ = .666	+ T	+ 21865	+ 1587	+ 1057			+ 1057
	- T	- 29153	- 2116	- 1409			- 1409
h = 2.33'							
	+ MOM	+ 44803					+35031
	- MOM	- 59070					+34320
G_1	D.L.	+ 3704	+20550	+15879	+ 18439	+ 11703	+27582
	+ C.L.L.	+ 10096	+ 614	+ 474	+ 5054	+ 3208	+ 3682
SIN φ = .6347	- C.L.L.	- 8372	+ 3760	+ 2905	+ 3926	+ 2492	+ 5397
COS φ = .7727	+ T	+ 9645	+ 1587	+ 1226			+ 1226
	- T	- 12860	- 2116	- 1635			- 1635
h = 2.14'							
	+ MOM	+ 23445					+32490
	- MOM	- 24936					+31344
G_2	D.L.	+ 1303	+20550	+17971	+12844	+ 6229	+24200
	+ C.L.L.	+ 20049	+ 2153	+ 1883	+ 4612	+ 2237	+ 4120
SIN φ = .485	- C.L.L.	- 9152	+ 4820	+ 4215	+ 2142	+ 1039	+ 5254
COS φ = .8745	+ T	+ 916	+ 1587	+ 1388			+ 1388
	- T	- 1222	- 2116	- 1850			- 1850
h = 1.97'							
	+ MOM	+ 22268					+ 29708
	- MOM	- 9071					+27604
G_3	D.L.	+ 1924	+20550	+19549	+ 7949	+ 2448	+21997
	+ C.L.L.	+ 20612	+ 3760	+ 3577	+ 3926	+ 1209	+ 4786
SIN φ = .308	- C.L.L.	- 10140	+ 3760	+ 3577	+ 1274	+ 392	+ 3969
COS φ = .9513	+ T	- 4614	+ 1587	+ 1510			+ 1510
	- T	+ 6152	- 2116	- 2013			- 2013
h = 1.81'							
	+ MOM	+ 28688					+ 24770
	- MOM	- 12830					+ 27476
G_4	D.L.	+ 1446	+20550	+20441	+ 3239	+ 334	+20775
	+ C.L.L.	+ 17400	+ 5474	+ 5445	+ 4186	+ 431	+ 5876
SIN φ = .103	- C.L.L.	- 7026	+ 4139	+ 4117	+ 1336	+ 137	+ 4254
COS φ = .9947	+ T	- 7272	+ 1587	+ 1578			+ 1578
	- T	+ 9696	- 2116	- 2105			- 2105
h = 1.66'							
	+ MOM	+ 28542					+24546
	- MOM	- 12852					+26600

TABLE XXIII COMPUTATION OF MAXIMUM STRESSES - CLEAR SPAN

99	100	101	102	103	104	105	106	107	108	109	110	111	112
POINT	MOMENT	NORMAL	X_o INCHES	$\frac{h}{X_o}$	$\frac{X_o}{h}$	p	n	$\frac{NX_o}{f_c b h^2}$	K	$\frac{h-d'}{Kh}-1$	$\frac{f_c b h}{N}$	f_c LB/IN²	f_s LB/IN²
O	+44803	35031	15.34	1.821		.0119	12	.135	.50	.855		425	4350
	-59070	34320	20.65	1.352		.0119	12	.136	.43	1.165		557	7800
G_1	+23445	32490	8.66	2.965		.013	12	.130	.70	.315		273	1034
	-24936	31344	9.55	2.689		.013	12	.130	.65	.420		291	1470
G_2	+22268	29708	8.99	2.630		.011	12	.125	.62	.480		318	1830
	- 9071	27604	3.94	5.995	.167	.011	12				1.72	168	COMP
G_3	+28688	24770	13.90	1.563		.012	12	.130	.45	.101		468	567
	-12830	27476	5.60	3.878		.012	12	.120	.82	.100		227	272
G_4	+28542	24546	13.97	1.427		.013	12	.135	.43	1.09		532	6960
	-12852	26600	5.80	3.436		.013	12	.125	.78	.154		259	479

$$X_o = \frac{M \times 12}{N} = \text{INCHES}$$

FOR TENSION OVER PART OF THE SECTION USE DIAG 5 AND $\frac{NX_o}{f_c b h^2}$

FOR COMPRESSION OVER ENTIRE SECTION USE DIAG 4 AND $\frac{f_c b h}{N}$

SPAN = 71.72 FT

RISE = 18.55 FT.

CHAPTER VI

ELASTIC PIER ANALYSIS

The analysis considering the yielding of the pier is based upon the theory presented in a paper by Mr. A. Hrennicoff[8] and published in the A. S. C. E. Transaction No. 101, year 1936. For a more detailed exposition of the theory the reader is referred to that paper. For the convenience of the reader the theory is presented here in outline.

Essentially the method consists of setting up expressions for the moment and thrust at the junction of the two arches and pier as functions of the rotation α, and displacement δ, of the junction. A rotation α of $\frac{I}{E}$ radians is considered and a displacement δ of $\frac{I}{E}$ feet. The effect of each is considered separately and the two resulting equations solved simultaneously.

An angular rotation of the junction without any displacement will cause a change in the moment and thrust acting on each of the three members meeting at the joint. Similarly, a horizontal displacement without any rotation will cause a change in the moment and thrust acting on each of the three members meeting at the joint. Loading either

[8] Op. Cit. page 388

arch rib will cause, in general, the application of both a moment and a thrust to the common junction. The distribution of this thrust and moment to the other two members meeting there will result in a change of the value computed by Fixed End Theory, the value assumed initially applied to the joint.

It would be possible to set up equations for the moment and thrust at the springing due to the application of a load to each division point on each arch, solve these simultaneously, and obtain the corrected values of each. A simpler procedure, however, would be to apply a unit moment to the joint and obtain the change in thrust and moment on each of the members meeting there, then to apply a unit thrust to the joint and obtain the change in moment and thrust on each of the members meeting there.

With these distributions known, we may proceed as follows: apply to the joint the Fixed End Moment due to any given load, obtain by proportion the change in moment and thrust due to the yielding of the joint, apply next to the joint the fixed end thrust, obtain by proportion the change in moment and thrust due to the yielding of the joint. The algebraic sum of these changes will give the total change in either moment or thrust, as desired, both at the junction point and also, using carry-over factors to be derived, at the far end of either arch. This method may be applied, using tabulation of values to load functions for any division point on the arches, and will be found less laborious.

For the derivation of the distribution and carry-over factors, we use the conventions: a moment which causes right hand rotation of the joint is designated as positive; a thrust which translates the joint to the right is designated as positive. The effect of the vertical yielding of the center pier is considered negligible and is not included in the analysis.

For arch rotation factors, we have the following symbols:

m_N = Moment exerted by the near end of the arch on the pier joint due to a rotation, without translation, of the joint equal to $\frac{I}{E}$ radians.

m_F = Moment exerted by the far end of the arch on the far joint due to a rotation, without translation, of the pier joint equal to $\frac{I}{E}$ radians.

h_N = Thrust exerted by the near end of the arch on the pier joint due to a rotation, without translation, of the pier joint equal to $\frac{I}{E}$ radians.

h_F = Thrust exerted by the far end of the arch on the far joint due to a rotation, without translation, of the pier joint equal to $\frac{I}{E}$ radians.

Rotation Factors For Arch

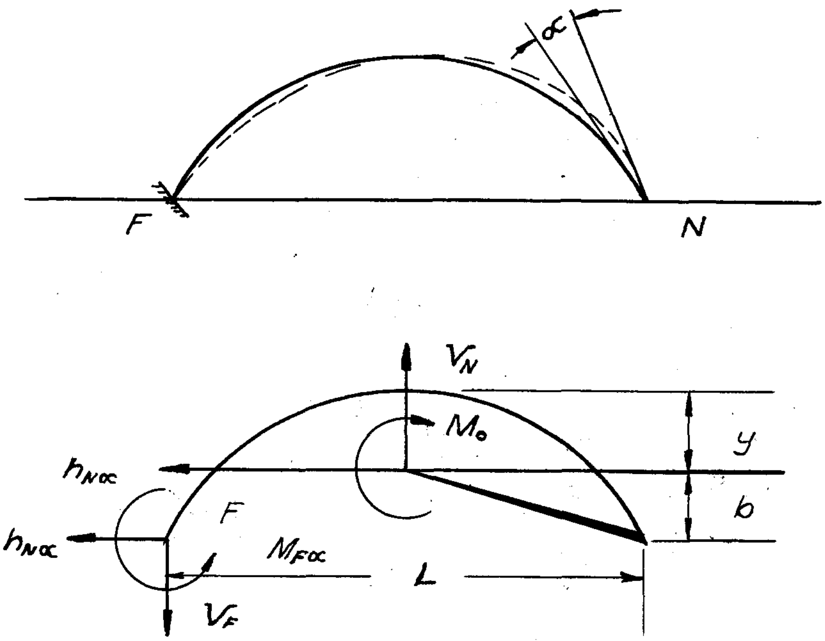


Fig. 22

We have from Fig. 22 that

$$m_N = -\left[M_O + V_N \frac{L}{2} - h_N \cdot b \right]$$

if the joint rotates without translation through an angle of $\frac{I}{E}$ we have the displacements of the rigid bracket at point O, (1) a rotation of $\frac{I}{E}$, (2) vertical displacement $\frac{L}{2E}$ upward, (3) horizontal displacement $\frac{b}{E}$ to the right.

hence

$$M_O = \frac{1}{\int \frac{ds}{I}}$$

$$V_N = \frac{L}{2E \int \frac{x^2\, ds}{I}}$$

$$h_{N\alpha} = \frac{b}{\int \frac{y^2 ds}{I} + \int \frac{dx \cos\phi}{A}} .$$

From this we have.

$$m_{N\alpha} = -\left[\frac{1}{\int \frac{ds}{I}} + \frac{(\frac{L}{2})^2}{\int \frac{x^2\, ds}{I}} + \frac{b^2}{\int \frac{y^2\, ds}{I} + \int \frac{ds \cos\phi}{A}} \right]$$

$$m_{F\alpha} = \frac{1}{\int \frac{ds}{I}} - \frac{(\frac{L}{2})^2}{\int \frac{x^2\, ds}{I}} + \frac{b^2}{\int \frac{y^2\, ds}{I} + \int \frac{dx \cos\phi}{A}}$$

$$h_{F\alpha} = - h_{N\alpha} = \frac{b}{\int \frac{y^2 ds}{I} + \int \frac{dx \cos\phi}{A}}$$

$$\text{also } b = \frac{\int \frac{y_1 ds}{EI}}{\int \frac{ds}{EI}} .$$

For Arch Translation Factors we have the following symbols:

$m_{N\sigma}$ = Moment exerted by the near end of the arch on the pier joint due to a translation, without rotation, of the pier joint equal to $\frac{I}{E}$ ft.

$m_{F\sigma}$ = Moment exerted by the far end of the arch on the far joint due to a translation without rotation, of the pier joint equal to $\frac{I}{E}$ ft.

$h_{n\delta}$ = Thrust exerted by the near end of the arch on the pier joint due to a translation, without rotation, of the pier joint equal to $\frac{I}{E}$ ft.

$h_{F\delta}$ = Thrust exerted by the far end of the arch on the far joint due to a translation, without rotation, of the pier joint equal to $\frac{I}{E}$ ft.

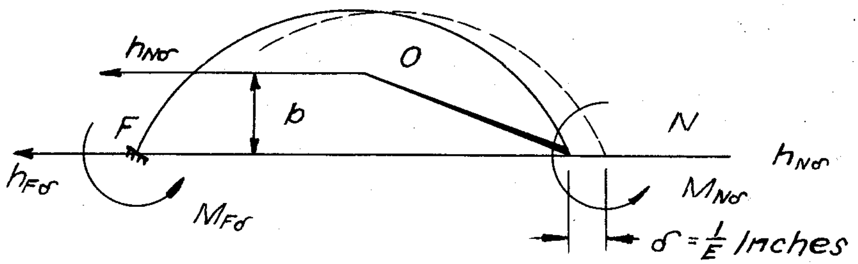


Fig. 23

From Fig. 23 we have. $h_{N\delta} = -\dfrac{1}{\int \frac{y^2 ds}{I} + \int \frac{dx \cos}{A}}$

also $h_{F\delta} = - h_{N\delta}$.

We have also $m_{F\delta} = -m_{N\delta} = \dfrac{b}{\int \frac{y^2 ds}{I} + \int \frac{dx \cos}{A}}$.

The above equations were derived supposing the left end of the arch to be fixed and the right end to be displaced, but may be used also when the right end is fixed and the left end is displaced.

Pier Factors

For the pier we have the following symbols;

m_T = Moment exerted on the joint by the pier when the pier top is rotated, without translation, through an angle of $\frac{I}{E}$ radians.

h_T = Thrust exerted on the joint by the pier when the pier top is rotated, without translation, through an angle of $\frac{I}{E}$ radians.

m_T = Moment exerted on the joint by the pier when the pier top is translated, without rotation, through a distance of $\frac{I}{E}$ ft.

h_T = Thrust exerted on the joint by the pier when the pier top is translated, without rotation, through a distance of $\frac{I}{E}$ ft.

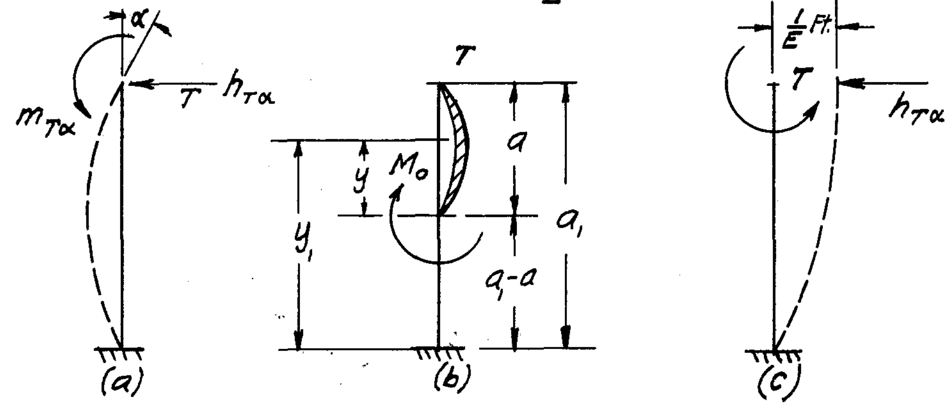


FIG 24

We assume the base of the pier to be fixed. Using the elastic center of the pier with a rigid arm connecting it to the pier top, we have the following equations:

$$m_{T\alpha} = -\left[\frac{1}{\int\frac{dy}{I}} - \frac{a^2}{\int\frac{y^2dy}{I}}\right]$$

$$h_{T\alpha} = \frac{a}{\int \frac{y^2 dy}{I}}$$

$$m_{T\delta} = \frac{a}{\int \frac{y^2 dy}{I}}$$

$$h_{T\delta} = -\frac{1}{\int \frac{y^2 dy}{I}}$$

The first step in the elastic pier analysis is the computation of the fixed-end moment, thrust, and vertical reactions for a unit load placed at the center of each arch division. This is merely a repetition of the first steps in the usual fixed-end analysis, and the five tables immediately following are exactly the same in form as the corresponding tables previously used in the fixed-end analysis. As explained elsewhere, the span for this analysis was slightly larger than that for the previously made fixed-end analysis.

The second step is the calculation of the elastic constants for the pier. These calculations are given in Table XXVIII and are practically self explanatory. The calculation of the elastic pier functions is next made in Table XXIX.

It will, of course, be understood that the integral signs used in the exposition of the theory in Chapter VI are replaced by summation signs in the actual calculation of the various constants. The calculation of the distribution factors is made on Page 134 by substitution into the formulas already given.

The actual distribution of an unbalanced moment and an unbalanced thrust when applied to the common junction is carried out in Tables XXX and XXXI. The unit used is 1000 ft lb for the moment and 1000 lb for the thrust. This rendered the arithmetical work somewhat easier to handle than would have been the case with the choice of a smaller unit.

The tabulation followed in the solution is the same as that given by Mr. Hrennicoff[9] in his original presentation. The finding of α and δ involves the solution, in each case, of two simultaneous equations. This has already been done, with the aid of a calculating machine, thus arriving at a high degree of numerical precision. Probably such precision is not really needed, but it does make possible a more satisfactory check on the correctness of the arithmetic.

Table XXXII, which follows, is an arrangement used by the writer. The signs are moment distribution signs. The operations are indicated by a series of abbreviations. F.E. means the fixed-end value as obtained by the fixed-end analysis. M.C. is used to designate the change in this value caused by the yielding of the pier under the application of the resulting fixed-end moment to the common junction. T.C. is used to designate the change due to the application of the accompanying fixed-end thrust to the common junction. The algebraic sum of all these -- the initial fixed-end value, the change caused by the thrust, and the change caused by the moment -- is placed opposite the summation sign and is the final value of the given function with the elastic yielding of the center pier considered.

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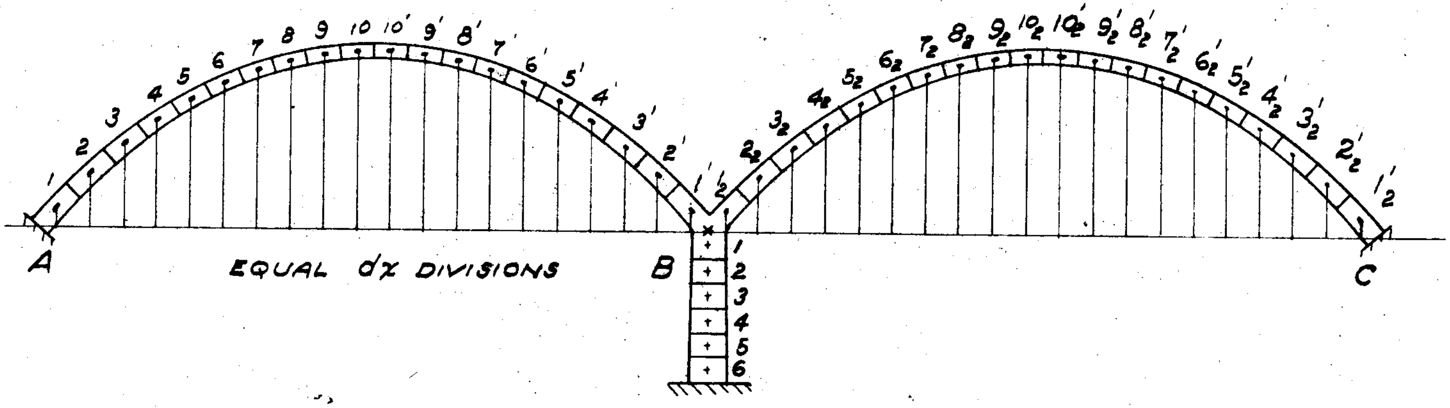
[9] Op. Cit. page 397

The vertical forces at the springing found by the fixed-end analysis are not correct for the elastic analysis. The correct values might have been found by a method similar to that used in the case of the moments and thrusts, but, instead, was found by computing the simple V (which could be done here, as both supports were on the same level), and modifying this by the change due to the moments at each end of the arch. These values are given in Table XXXIII.

The calculation of influence ordinates was carried on from this point in the usual manner. Since such calculations represent a considerable labor, they were made for only two girder points, namely, those near the crown. The fixed-end analysis previously made indicated that these and the springing points were the only critical ones. The influence curves for these critical points are shown on Figures 27 and 28. The final compilation of maximum moments and thrusts was made with the aid of Figure 29, which indicates the position of wheel loads for producing the greatest stresses in the arch ring. The dead load and the temperature values were taken from the fixed end analysis previously made. The calculation of maximum fiber stresses in concrete and steel for the arch points considered is shown in Table XXXIII. The concrete stresses show a small increase, as would, of course, be expected. The amount of the increase is about 50 lb per square inch, which is rather small, considering the fact that both spans are loaded to produce this. The final maximum concrete stresses are well within the safe allowable value.



CENTERLINE SPAN 74'-0"

FIG 25

NUMBERING SYSTEM
FOR
ELASTIC PIER ANALYSIS

TABLE XXIV COMPUTATION OF ELASTIC WEIGHTS

1	2	3	4	5	6	7	8	9	10	11
POINTS	$h = A$	h^3	$I_c = \frac{h^3}{12}$	$\frac{h}{2}$	$\frac{h}{2} - d'$	$\left[\frac{h}{2} - d'\right]^2$	$I_s = \left[\frac{h}{2} - d'\right]^2 nA_s$	$I = I_c + I_s$	ds	$\frac{ds}{I} = \Delta$
0										
1	2.33	12.650	1.054	1.165	.995	.9900	.3025	1.356	5.53	4.08
2	2.24	11.24	.9365	1.120	.950	.9025	.2755	1.212	5.13	4.23
3	2.14	9.800	.8166	1.070	.900	.8100	.2475	1.064	4.78	4.49
4	2.06	8.742	.7285	1.030	.860	.7396	.2260	.955	4.49	4.71
5	1.98	7.762	.6470	.990	.820	.6724	.1603	.807	4.26	5.28
6	1.90	6.859	.5715	.950	.780	.6084	.1450	.716	4.07	5.68
7	1.83	6.128	.5107	.915	.745	.5550	.1322	.643	3.93	6.11
8	1.76	5.451	.4542	.880	.710	.5041	.1200	.574	3.82	6.66
9	1.69	4.827	.4022	.845	.675	.4556	.1085	.511	3.74	7.32
10	1.62	4.251	.3542	.810	.640	.4096	.0977	.452	3.70	8.19
									$\frac{1}{2}\Sigma\Delta$	56.75

SPAN = 74.00

RISE = 19.87

TABLE XXV COMPUTATIONS OF V_0

	12	13	14	15	16	17	18
POINTS	$Z-20$ WHERE $Z=\frac{2X}{dX}$	$Q=(Z-20)\Delta$	$-\sum_a^1 (Z-20)\Delta$	$-\frac{1}{2}\sum_a^1 (Z-\frac{2a}{\partial x})Q$	$Z^2+(40-Z)^2$	$[Z^2+(40-Z)^2]\Delta$	V_0
0							
1	-19	-77.52	77.52	6140.51	1522	6209.76	1.000000
2	-17	-71.91	149.43	6062.99	1378	5828.94	.987375
3	-15	-67.35	216.78	5913.56	1250	5612.50	.963040
4	-13	-61.23	278.01	5696.78	1138	5359.98	.927737
5	-11	-58.08	336.09	5418.77	1042	5501.76	.882462
6	-9	-51.12	387.21	5082.68	962	5464.16	.827729
7	-7	-42.77	429.98	4695.47	898	5486.78	.764671
8	-5	-33.30	463.28	4265.49	850	5661.00	.694647
9	-3	-21.96	485.24	3802.21	818	5987.76	.619201
10	-1	- 8.19	493.43	3316.97	802	6568.38	.540178
		493.43	3316.97		$\Sigma z^2\Delta$	57681.02	

$$V_0 = \frac{-\frac{1}{2}\sum_a^1 (Z-\frac{2a}{\partial x})(Z-20)\Delta}{F}$$

SPAN = 74.00
RISE = 19.87

TABLE XXVI COMPUTATIONS OF H_0

19	20	21	22	23	24	25	26	27	28	29	30
POINTS	Y	Δ	$Y\Delta$	$Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}$	$R=\Delta\left[Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right]$	$\Sigma_a^1\Delta\left[Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right]$	$\frac{1}{2}\Sigma_a^1\left[Z-\frac{2a}{dx}\right]R$	$Y\Delta\left[Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right]$	$\cos\phi$	$\frac{\cos\phi}{A}$	H_0
0				-14.745					.660		
1	2.076	4.08	8.47008	-12.669	-51.68952	- 51.68952	0.0000	-107.30744	.670	.2875	.0000
2	5.940	4.23	25.12620	- 8.805	-37.24515	-88.93467	- 51.68952	-221.23619	.723	.3225	.0555
3	9.200	4.49	41.30800	- 5.545	-24.89705	-113.83172	-140.62419	-229.05386	.774	.3615	.1511
4	12.000	4.71	56.52000	- 2.745	-12.92895	-126.76067	-254.45591	-155.1474	.824	.3995	.2734
5	14.300	5.28	75.50400	- .445	- 2.34960	-129.11027	-381.21658	- 33.59928	.870	.4390	.4096
6	16.188	5.68	91.94784	+ 1.443	8.19624	-120.91403	-510.3268	132.68073	.9085	.4780	.5484
7	17.680	6.11	108.02480	2.935	17.93285	-102.98118	-631.2409	317.05279	.944	.5160	.6783
8	18.770	6.66	125.00820	4.025	26.8065	- 76.17468	-734.2221	503.15800	.972	.5515	.7889
9	19.476	7.32	142.56432	4.731	34.63092	-41.54376	-810.3967	674.47180	.990	.5860	.8708
10	19.820	8.19	162.32580	5.075	41.56425	+ .0204	-851.9405	823.80344	.999	.6160	.9154
CR	19.87	56.75	836.79924		.02049			1704.823		4.5575	

$$H_0=\frac{-\frac{1}{2}\Sigma_a^1\left[Z-\frac{2a}{dx}\right]\left[Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right]\Delta}{C}$$

$$F=\frac{1}{2}\Sigma Z^2\Delta-200\,\Sigma\Delta=6140.51$$

$$Z=\frac{2x}{dx}=$$

$$\frac{\Sigma Y\Delta}{\Sigma\Delta}=14.745$$

$$dx=\frac{l}{20}=3.70'$$

$$C=\frac{1}{dx}\Sigma_a^2 Y\Delta\left[Y-\frac{\Sigma Y\Delta}{\Sigma\Delta}\right]+\Sigma\frac{\cos\phi}{A}=930.636$$

$$H_t=\frac{20\,etE}{C}=46.421\dagger$$

SPAN = 74.00

RISE = 19.87

TABLE XXVII COMPUTATIONS FOR M_o

31	32	33	34	35	36	37	38	39	40	41
POINT	Z	H_o	V_o	Δ	$\sum_a^l \Delta$	$\frac{1}{2}\sum_a^l(Z-\frac{2a}{dx})\Delta$	$\frac{dx}{\Sigma\Delta}$(COL 37)	$\frac{\Sigma Y\Delta}{\Sigma\Delta}H_o$	$-20\frac{dx}{2}V_o$	M_o
0	0	0	0							
1	1	0	1.0000	4.08	109.42	1078.25	35.1500	.00	37.000	- 1.8500
2	3	.0555	.98737	4.23	105.19	968.83	31.58298	.818347	36.53269	- 4.13131
3	5	.1511	.96304	4.49	100.70	863.64	28.15388	2.22797	35.63248	- 5.25063
4	7	.2734	.927737	4.71	95.99	762.94	24.87115	4.03128	34.32627	- 5.42384
5	9	.4096	.88246	5.28	90.71	666.95	21.74197	6.03955	32.65102	- 4.86950
6	11	.5484	.82773	5.68	85.03	576.24	18.7849	8.086158	30.62601	- 3.75495
7	13	.6783	.764671	6.11	78.92	491.21	16.0130	10.00153	28.29283	- 2.27830
8	15	.7889	.694647	6.66	72.26	412.29	13.44028	11.63233	25.70194	- .62933
9	17	.8708	.619200	7.32	64.94	340.03	11.08467	12.83995	22.91040	1.01422
10	19	.9154	.540178	8.19	56.75	275.09	8.96768	13.49757	19.98658	2.47867
10'	19	.9154	.45982	8.19	48.56	218.34	7.11768	13.49757	17.01334	3.60191
9'	17	.8708	.38080	7.32	41.24	169.78	5.53467	12.83995	14.08960	4.2850
8'	15	.7889	.30535	6.66	34.58	128.54	4.19028	11.63233	11.29806	4.52456
7'	13	.6783	.23533	6.11	28.47	98.96	3.06301	10.00153	8.70721	4.35733
6'	11	.5484	.17227	5.68	22.79	65.49	2.13491	8.086158	6.37399	3.84708
5'	9	.4096	.11754	5.28	17.51	42.70	1.39198	6.03955	4.34898	3.08255
4'	7	.2734	.07226	4.71	12.80	25.19	.82117	4.03128	2.67373	2.17872
3'	5	.1511	.03696	4.49	8.31	12.39	.40390	2.22797	1.36752	1.26435
2'	3	.0555	.01263	4.23	4.08	4.08	.13300	.818347	.46731	.48404
1'	1	.00	.000	4.08	.0	.0	.000	.00	.00	.000
0'	0	0	.000							
		Σ 9.3828	10.00		1078.25	7195.94	234.5811	138.3494	369.99996	+ 2.93056

$$M_o = \frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l\left(Z-\frac{2a}{dx}\right)\Delta + H_o\frac{\Sigma Y\Delta}{\Sigma\Delta} - 20\frac{dx}{2}V_o$$

$$\frac{dx}{\Sigma\Delta} = .0325991 \qquad -20\frac{dx}{2} = -37.00$$

$$\frac{\frac{1}{2}\text{Sum } 22}{\frac{1}{2}\text{Sum } 11} \times \text{Sum } 33 = \text{Sum } 39$$

$$-20\frac{dx}{2}(\text{Sum } 34) = \text{Sum } 40$$

Sum 32 + Sum 39 + Sum 40 = Sum 41

SPAN = 74.00 FT.

RISE = 19.87 FT.

TABLE XXVIII ELASTIC PIER CONSTANTS

42	43	44	45	46	47	48
DIV. NO.	THICK FEET d	$\frac{d}{2}$	$\frac{d}{2}-.17$	$(\frac{d}{2}-.17)^2$ $x(n-1)A_s$	$I_c = \frac{bd^3}{12}$	$I = I_c + I_s$
1	4.110	2.055	1.885	.846	5.76	6.606 Ft^4
2	4.185	2.093	1.923	.881	6.11	6.991
3	4.265	2.133	1.963	.918	6.48	7.398
4	4.350	2.175	2.005	.958	6.86	7.818
5	4.425	2.213	2.043	.996	7.23	8.226
6	4.500	2.250	2.080	1.030	7.60	8.63

TABLE XXIX ELASTIC PIER FUNCTIONS

49	50	51	52	53	54	55	56
DIV. NO.	y_1	$\frac{\Delta y}{I}$	$y_1 \frac{\Delta y}{I}$	$y = y_1 - \bar{Y}$	$y \frac{\Delta y}{I}$	y^2	$y^2 \frac{\Delta y}{I}$
1	15	.454	6.810	7.0319	+3.19248	49.4476	22.4921
2	12	.4285	5.141	4.0319	+1.72767	16.2562	6.96578
3	9	.406	3.654	1.0319	+.41895	1.06481	.43231
4	6	.384	2.304	-1.9681	-.75575	3.87341	1.48739
5	3	.365	1.095	-4.9681	-1.81335	24.6820	9.00893
6	0	.3475	.000	-7.9681	-2.76891	63.4906	22.06298
		$\Sigma = 2.3850$	19.004		$\Sigma = .0011$		$\Sigma = 62.40660$

$$\bar{Y} = \frac{\Sigma y_1 \frac{\Delta y}{I}}{\Sigma \frac{\Delta y}{I}}$$

$$a = 16.500 - 7.9681 = 8.5319 \text{ Ft}$$

CALCULATION OF DISTRIBUTION CONSTANTS

FOR THE ARCH

$$\int \frac{y^2 ds}{I} = 2\sum \text{Col } 27 \text{ (From Table XXVI)} = 3409.6 \text{ Use } 3410. \left(\frac{1}{Ft}\right)$$

$$\int \frac{x^2 ds}{I} = 2 \times F \times \left(\frac{\Delta X}{2}\right)^2 \text{(From Table XXVI)} = 42031.8 \text{ Use } 42030. \left(\frac{1}{Ft}\right)$$

$$\int \frac{ds}{I} = 2 \times \Sigma \Delta \text{ (From Table XXIV)} = 113.5 \left(\frac{1}{Ft^3}\right)$$

$$\int \frac{dx \cos\theta}{A} = 3.7 \times 9.115 \text{ (From Table XXVI)} = 33.72 \text{ Use } 34. \left(\frac{1}{Ft}\right)$$

FOR THE PIER

$$\int \frac{dy}{I} = \text{(From Table XXIX)} = 2.3850 \left(\frac{1}{Ft^3}\right)$$

$$\int \frac{y^2 dy}{I} = \text{(From Table XXIX)} = 62.4066 \left(\frac{1}{Ft}\right)$$

THESE VALUES ARE SUBSTITUTED INTO THE EQUATIONS GIVEN IN CHAPTER VI THUS OBTAINING THE DISTRIBUTION FACTORS GIVEN ON THE FOLLOWING SHEET.

CALCULATION OF DISTRIBUTION FACTORS

$$m_{N\alpha} = \left[\frac{1}{113.5} + \frac{(37)^2}{42030} + \frac{(14.745)^2}{3410+34}\right] = -.104511$$

$$m_{F\alpha} = \left[\frac{1}{113.5} - \frac{(37)^2}{42030} + \frac{(14.745)^2}{3410+34}\right] = +.0393671$$

$$h_{N\alpha} = -\frac{14.745}{3410+34} = -.00428136$$

$$h_{F\alpha} = +.00428136$$

$$m_{N\delta} = -\frac{14.745}{3444} = -.00428136$$

$$m_{F\delta} = +.00428136$$

$$h_{N\delta} = -\frac{1}{3444} = -.00029036$$

$$h_{F\delta} = +.00029036$$

$$m_{T\alpha} = -\left[\frac{1}{2.385} + \frac{(8.5319)^2}{62.4066}\right] = -1.58572$$

$$m_{T\delta} = \frac{8.5319}{62.4066} = +.13671$$

$$h_{T\alpha} = \frac{8.5319}{62.4066} = +.13671$$

$$h_{T\delta} = -\frac{1}{62.4066} = -.016024$$

TABLE XXX DISTRIBUTION OF MOMENT AT "B"

57	58	59	60	61	62	63	64	65	66	67	68
ITEM	CONDITION	A_B	B_A		$B_{B'}$		B_C		TOTAL, JOINT B		C_B
		m	h	m	h	m	h	m	h	m	m
1	$\alpha = \frac{1}{E}$	.039367	-.004281	-.10451	.1367	-1.586	-.004281	-.10451	.128 37	-1.79502	.039367
2	$\delta = \frac{1}{E}$	.004281	-.000290	-.004281	-.01602	.1367	-.000290	-.004281	-.0166007	.128137	.004281
3	FIX. COND.	0	0	0	0	0	0	0	0	1000	0
4	$\alpha = \frac{1240.75}{E}$	48.8448	-5.31217	-129.671	169.6113	-1967.83	-5.31217	-129.671	158.9867	-2227.181	48.8448
5	$\delta \ \frac{9577.16}{E}$	41.00365	-2.78082	-41.003	-153.4261	1309.19	-2.78082	-41.003	-158.9875	1227.188	41.0036
6	TOTALS	89.8484	-8.09299	-170.675	16.1852	-658.64	-8.09299	-170.675	Σ 3,4,5	Σ 3,4,5	89.8484

$\alpha = 1240.756$

$\delta = 9577.16$

1) $h_\alpha \alpha + h_\delta \delta + hF = 0$

2) $m_\alpha \alpha + m_\delta \delta + mF = 0$

$$.128137\alpha - .0166007\delta + 0 = 0$$
$$-1.79502\alpha + .128137\delta + 1000 = 0$$

$$\alpha - .129554\,\delta = 0$$
$$-\alpha + .0713847\,\delta + 557.097 = 0$$
$$\delta = 9577.16$$

$$7.71877\alpha - \delta = 0$$
$$-14.00860\alpha + \delta + 7804.147 = 0$$
$$\alpha = 1240.756$$

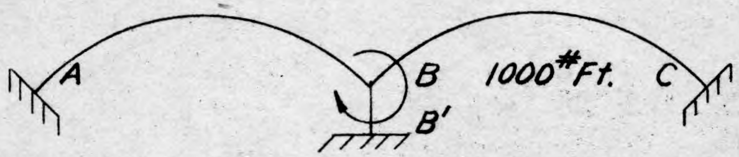


TABLE XXXI DISTRIBUTION OF THRUST AT B

69	70	71	72	73	74	75	76	77	78	79	80
ITEM	CONDITION	A_B	B_A		$B_{B'}$		B_C		TOTAL, JOINT B		C_B
		m	h	m	h	m	h	m	h	m	m
1	$\alpha = \frac{1}{E}$	.039367	-.004281	-.10451	.1367	-1.586	-.004281	-.10451	.128137	-1.79502	.039367
2	$\delta = \frac{1}{E}$	.004281	-.000290	-.004281	-.01602	.1367	-.000290	-.004281	-.016600	.128137	.0042814
3	FIX COND.	0	0	0	0	0	0	0	1000	0	0
4	$\alpha = \frac{9577.11}{E}$	377.022	-41.0034	-1000.90	1309.19	-15189.29	-41.0034	-1000.90	1227.182	-17191.104	377.022
5	$\delta = \frac{134162}{E}$	574.4012	-38.9553	-574.40	-2149.27	18339.94	-38.9553	-574.40	-2227.183	17191.116	574.401
6	TOTALS	951.423	-79.9587	-1575.30	-840.08	3150.65	-79.9587	-1575.30	Σ 3,4,5	Σ 3,4,5	951.423

$\alpha = 9577.111$

$\delta = 134162$

1) $h_\alpha \alpha + h_\delta \delta + hF = 0$

2) $m_\alpha \alpha + m_\delta \delta + mF = 0$

$$.128137\alpha - .0166007\delta + 1000 = 0$$
$$-1.79502\alpha + .128137\delta + 0 = 0$$

$$\alpha - .129554\delta + 7804.147 = 0$$
$$-\alpha \quad .0713847\delta + 0 = 0$$

$$\delta = 134162$$

$$7.71877\alpha - \delta + 60238.4 = 0$$
$$-14.00860\alpha + \delta = 0$$

$$\alpha = 9577.111$$

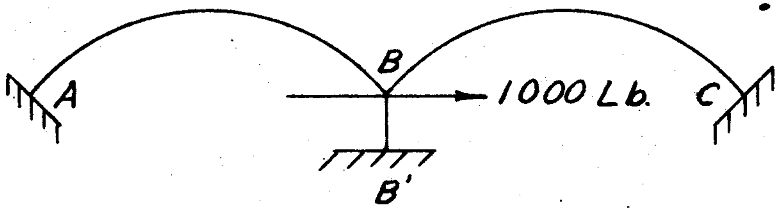


TABLE XXXII DISTRIBUTION OF MOMENTS AND THRUSTS

81	82	83	84	85	86	87	88
PT		MOMENT				THRUST	
		A_B	B_A	B_C	C_B	B_A	B_C
1	F.E.	+1.8500	.00	.00	.00	.00	.00
	M.C.	.0000	.00	.00	.00	.00	.00
	T.C.	.0000	.00	.00	.00	.00	.00
	Σ	+1.8500	.00	.00	.00	.00	.00
2	F.E.	+4.1313	+ .4840	.00	.00	+ .0555	.00
	M.C.	+ .0435	- .0826	- .0826	+ .0435	- .0039	- .0039
	T.C.	+ .0528	- .0874	- .0784	+ .0528	- .0044	- .0044
	Σ	+4.2276	+ .3140	- .1700	+ .0963	+ .0471	- .0083
3	F.E.	+5.2506	+1.2643	.00	.00	+ .1511	.00
	M.C.	+ .1136	- .2158	- .2158	+ .1136	- .0102	- .0102
	T.C.	+ .1438	- .2380	- .2380	+ .1437	- .0121	- .0121
	Σ	+5.5080	+ .8105	- .4538	+ .2573	+ .1288	- .0223
4	F.E.	+5.4238	+2.1787	.00	.00	+ .2734	.00
	M.C.	+ .1957	- .3718	- .3718	+ .1957	- .0176	- .0176
	T.C.	+ .2601	- .4307	- .4307	+ .2601	- .0218	- .0218
	Σ	+5.8797	+1.3762	- .8025	+ .4558	+ .2339	- .0395
5	F.E.	+4.8695	+3.0825	.00	.00	+ .4096	.00
	M.C.	+ .2769	- .5261	- .5261	+ .2769	- .0249	- .0249
	T.C.	+ .3896	- .6452	- .6452	+ .3897	- .0327	- .0327
	Σ	+5.5361	+1.9112	-1.1713	+ .6666	+ .3519	- .0577
6	F.E.	+3.7549	+3.8471	.00	.00	+ .5484	.00
	M.C.	+ .3456	- .6566	- .6566	+ .3456	- .0311	- .0311
	T.C.	+ .5217	- .8639	- .8639	+ .5217	- .0438	- .0438
	Σ	+4.6223	+2.3266	-2.3266	+ .8674	+ .4734	- .0750
7	F.E.	+2.2783	+4.3575	.00	.00	+ .6783	.00
	M.C.	+ .3915	- .7437	- .7437	+ .3915	- .0352	- .0352
	T.C.	+ .6453	-1.0685	-1.0685	+ .6453	- .0542	- .0542
	Σ	+3.3151	+2.5451	-1.8122	+1.0368	+ .5888	- .0895
8	F.E.	+ .6293	+4.5245	.00	.00	+ .7889	.00
	M.C.	+ .4065	- .7722	- .7722	+ .4065	- .0366	- .0366
	T.C.	+ .7505	-1.2427	-1.2427	+ .7506	- .0630	- .0630
	Σ	+1.7864	+2.1819	-2.0150	+1.1571	+ .6892	- .0997
9	F.E.	-1.0142	+4.2850	.00	.00	+ .8708	.00
	M.C.	+ .3850	- .7313	- .7313	+ .3850	- .0347	- .0347
	T.C.	+ .8285	-1.1371	-1.3718	+ .8285	- .0696	- .0696
	Σ	+ .1993	+2.1819	-2.1031	+1.2135	+ .7665	- .1043
6_A	F.E.	-1.6700	+4.0300	.00	.00	+ .8930	.00
	M.C.	+ .3621	- .6878	- .6878	+ .3621	- .0326	- .0326
	T.C.	+ .8496	-1.4067	-1.4067	+ .8496	- .0714	- .0714
	Σ	- .4583	+1.9355	-2.0945	+1.2117	+ .7890	- .1040
10	F.E.	-2.4786	+3.6019	.00	.00	+ .9154	.00
	M.C.	+ .3236	- .6147	- .6147	+ .3236	- .0291	- .0291
	T.C.	+ .8709	-1.4420	-1.4420	+ .8709	- .0732	- .0732
	Σ	-1.2841	+1.5451	-2.0568	+1.1945	+ .8130	- .1023

TABLE XXXII DISTRIBUTION OF MOM. & THRUSTS CONT'D.

89	90	91	92	93	94	95	96
PT		MOMENT				THRUST	
		A_B	B_A	B_C	C_B	B_A	B_C
10'	F.E.	-3.6019	+2.4786	.00	.00	+ .9154	.00
	M.C.	+ .2227	- .4230	- .4230	+ .2227	.0200	- .0200
	T.C.	+ .8709	-1.4420	-1.4420	+ .8709	- .0732	- .0732
	Σ	-2.5083	+ .6136	-1.8650	+1.0936	+ .8221	- .0932
G'_4	F.E.	-4.0300	+1.6700	.00	.00	+ .8930	.00
	M.C.	+ .1500	- .2850	- .2850	+ .1500	- .0135	- .0135
	T.C.	+ .8496	-1.4076	-1.4076	+ .8496	- .0714	- .0714
	Σ	-3.0303	- .0217	-1.6926	+ .9996	+ .8081	- .0849
9'	F.E.	-4.2850	+1.0142	.00	.00	+ .8708	.00
	M.C.	+ .0911	- .1731	- .1731	+ .0911	- .0082	- .0082
	T.C.	+ .8285	-1.3718	-1.3718	+ .8285	- .0696	- .0696
	Σ	-3.3654	- .5307	-1.5449	+ .9195	+ .7930	- .0778
8'	F.E.	-4.5245	- .6293	.00	.00	+ .7889	.00
	M.C.	- .0565	+ .1074	+ .1074	- .0565	+ .0051	+ .0051
	T.C.	+ .7505	-1.2427	-1.2427	+ .7506	- .0631	- .0631
	Σ	-3.8305	-1.7647	-1.1353	+ .6940	+ .7309	- .0580
7'	F.E.	-4.3573	-2.2783	.00	.00	+ .6783	.00
	M.C.	- .2047	+ .3885	+ .3885	- .2047	+ .0184	+ .0184
	T.C.	+ .6453	-1.0685	-1.0685	+ .6453	- .0542	- .0542
	Σ	-3.9167	-2.9583	- .6800	+ .4406	+ .6424	- .0358
6'	F.E.	-3.8471	-3.7549	.00	.00	+ .5484	.00
	M.C.	- .3374	+ .6409	+ .6409	- .3374	+ .0304	+ .0304
	T.C.	+ .5217	- .8639	- .8639	+ .5217	- .0438	- .0438
	Σ	-3.6627	-3.9780	- .2230	+ .1844	+ .5349	- .0134
5'	F.E.	-3.0825	-4.8695	.00	.00	+ .4096	.00
	M.C.	- .4375	+ .8311	+ .8311	- .4375	+ .0394	+ .0394
	T.C.	+ .3897	- .6452	- .6452	+ .3897	- .0327	- .0327
	Σ	-3.1303	-4.6836	+ .1858	- .0478	+ .4162	+ .0066
4'	F.E.	-2.1787	-5.4238	.00	.00	+ .2734	.00
	M.C.	- .4873	+ .9257	+ .9257	- .4873	+ .0439	+ .0439
	T.C.	+ .2601	- .4307	- .4307	+ .2601	- .0218	- .0218
	Σ	-2.4059	-4.9288	+ .4950	- .2272	+ .2954	+ .0220
3'	F.E.	-1.2643	-5.2506	.00	.00	+ .1511	.00
	M.C.	- .4717	+ .8961	+ .8961	- .4717	+ .0425	+ .0425
	T.C.	+ .1437	- .2380	- .2380	+ .1437	- .0121	- .0121
	Σ	-1.5923	-4.5925	+ .6581	- .3280	+ .1815	+ .0304
2'	F.E.	- .4840	-4.1313	.00	.00	+ .0555	.00
	M.C.	- .3712	+ .7051	+ .7051	- .3712	+ .0334	+ .0334
	T.C.	+ .0528	- .0874	- .0874	+ .0528	- .0044	- .0044
	Σ	- .8024	-3.5136	+ .6197	- .3184	+ .0844	+ .0289
1'	F.E.	.00	-1.8500	.00	.00	.00	.00
	M.C.	- .1662	+ .3157	+ .3157	- .1662	+ .0150	+ .0150
	T.C.	+ .00	.00	.00	+ .00	.00	.00
	Σ	- .1662	-1.5342	+ .3157	- .1662	+ .0150	+ .0150

TABLE XXXIII COMP. OF V IN NEAR SPAN "AB"

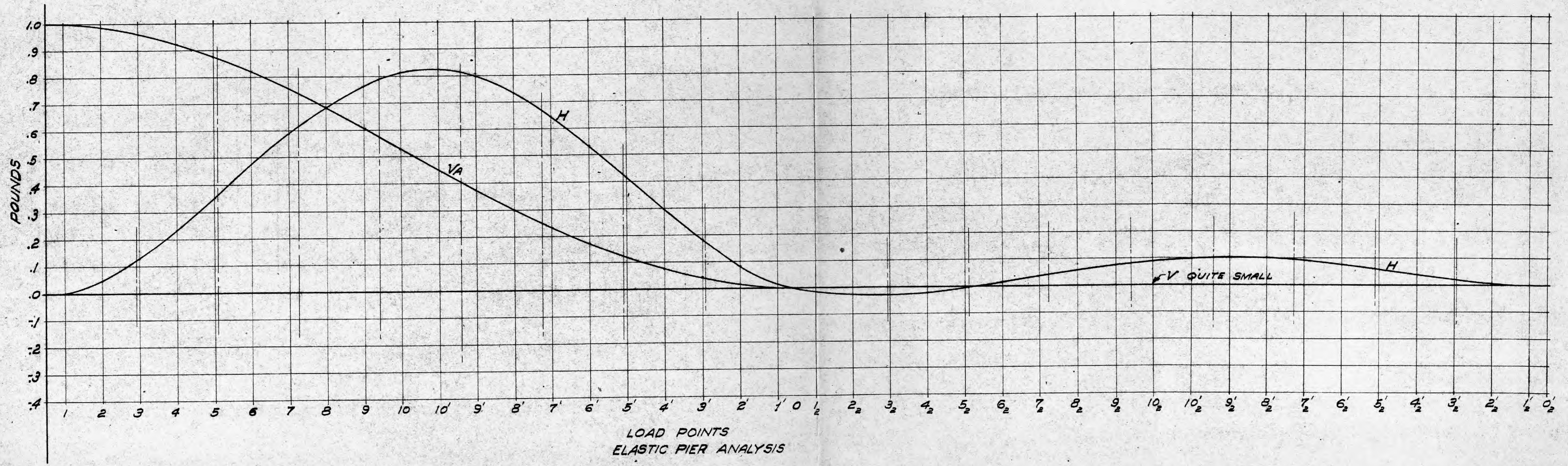
97	98	99	100	101	102	103
POINT	V_A SIMPLE	CHANGE IN V_A DUE TO MOMENTS	TOTAL V_A	V_B SIMPLE	CHANGE IN V_B DUE TO MOMENTS	TOTAL V_B
0						
1	.9750	.0250	1.0000	.0250	-.0250	0
2	.9250	.0614	.9864	.0750	-.0614	.0136
3	.8750	.0854	.9604	.1250	-.0854	.0396
4	.8250	.0980	.9230	.1750	-.0980	.0770
5	.7750	.1006	.8756	.2250	-.1006	.1244
6	.7250	.0939	.8189	.2750	-.0939	.1811
7	.6750	.0792	.7542	.3250	-.0792	.2458
8	.6250	.0580	.6830	.3750	-.0580	.3170
9	.5750	.0322	.6072	.4250	-.0322	.3928
10	.5250	.0035	.5285	.4750	-.0035	.4715
10'	.4750	-.0256	.4494	.5250	+.0256	.5506
9'	.4250	-.0526	.3724	.5750	.0526	.6276
8'	.3750	-.0756	.2994	.6250	.0756	.7006
7'	.3250	-.0929	.2321	.6750	.0929	.7679
6'	.2750	-.1032	.1718	.7250	.1032	.8282
5'	.2250	-.1056	.1194	.7750	.1056	.8806
4'	.1750	-.0991	.0759	.8250	.0991	.9241
3'	.1250	-.0836	.0414	.8750	.0836	.9586
2'	.0750	-.0583	.0167	.9250	.0583	.9833
1'	.0250	-.0230	.0020	.9750	.0230	.9980
0						

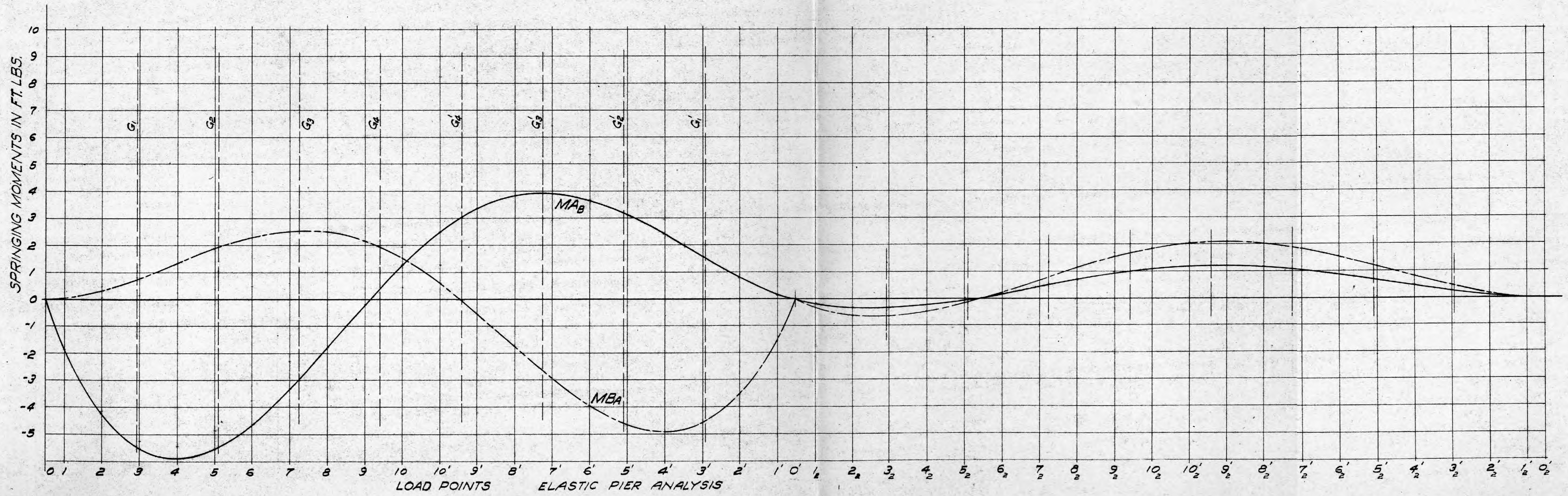
TABLE XXXIV
V FAR SPAN

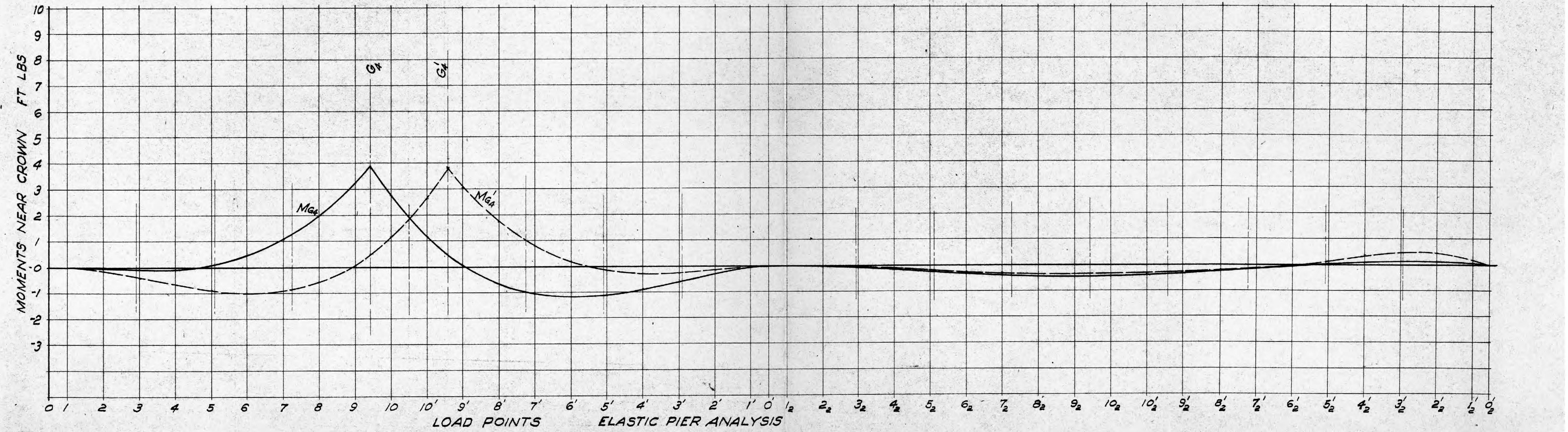
104.	105	106
PT.	M_A+M_B	V_A
0		
1_2	0	0
2_2	.07375	.0009966
3_2	.19646	.002655
4_2	.34666	.004684
5_2	.50475	.006821
6_2	.65308	.008825
7_2	.77536	.01048
8_2	.85789	.01159
9_2	.88960	.012021
10_2	.86223	.011652
$10_2'$	.77144	.01042
$9_2'$	.62540	.008451
$8_2'$	.44131	.005963
$7_2'$	.23939	.003235
$6_2'$	.03863	.000522
$5_2'$	-.13805	-.0018655
$4_2'$	-.26783	-.003619
$3_2'$	-.33012	-.004461
$2_2'$	-.30129	-.004071
$1_2'$	-.14955	-.002021
0		

TABLE XXXV SPRINGING MOM. AT A AND B-NET OR ADJUSTED

107	108	109	110	111	112	113	114	115
PT.	M_{OA}	$V_O \times 1.14$	$H_O \times 1.285$	M_{SP}	M_{OB}	$V_O \times 1.14$	$H_O \times 1.285$	M_{SP}
0								
1	-1.850	1.1400	0	-.710	0	0	0	0
2	-4.227	1.1245	-.06026	-3.1627	.3140	.01550	-.05749	.272
3	-5.508	1.0948	-.1655	-4.578	.8105	.04514	-.16550	.6901
4	-5.880	1.0522	-.30056	-5.128	1.376	.08778	-.30057	1.1632
5	-5.536	.99818	-.45219	-4.990	1.911	.14181	-.45219	1.6006
6	-4.622	.9335	-.60832	-4.297	3.326	.20645	-.60834	2.924
7	-3.315	.85978	-.75660	-3.212	2.545	.28021	-.75662	2.0686
8	-1.786	.77862	-.8856	-1.893	2.510	.36138	-.8857	1.9857
9	-.1993	.6922	-.98495	-.4920	2.182	.44779	-.98494	1.645
10	1.284	.6025	-1.04478	.8417	1.545	.53751	-1.04478	1.0377
10″	2.508	.51231	-1.05640	1.964	.6136	.51231	-1.05646	.0695
9′	3.365	.42453	-1.0190	2.770	-.5307	.42453	-1.01896	-1.125
8′	3.830	.34131	-.93920	3.232	-1.7646	.34131	-.93923	-2.3625
7′	3.917	.26459	-.82561	3.356	-2.958	.26459	-.82557	-3.5190
6′	3.663	.19585	-.68747	3.171	-3.978	.19585	-.687410	-4.4695
5′	3.130	.136116	53481	2.731	-4.683	.13611	-.534881	-5.0817
4′	2.406	.08526	-.37959	2.111	-4.929	.08652	-.37964	-5.222
3′	1.592	.047196	-.23322	1.506	-4.592	.047196	-.23325	-4.7780
2′	.802	.019038	-.0853	.7126	-3.513	.019038	-.10853	-3.6024
1′	.166	.00228	.019275	.2590	-1.534	.00228	-.019275	-1.551
0								


POUNDS
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
H
VA
V QUITE SMALL
H
1 2 3 4 5 6 7 8 9 10 10' 9' 8' 7' 6' 5' 4' 3' 2' 1' 0
LOAD POINTS
ELASTIC PIER ANALYSIS


SPRINGING MOMENTS IN FT. LBS.
LOAD POINTS
ELASTIC PIER ANALYSIS
MAB
MBA
G1
G2
G3
G4
G'4
G'3
G'2
G'1



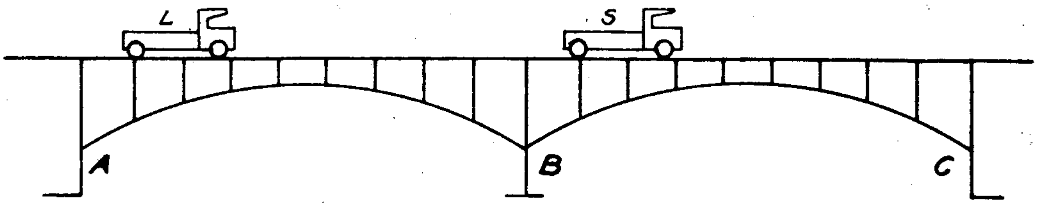
MOMENTS NEAR CROWN FT LBS
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
G_4
G_4'
M_{G4}
M_{G4}'
0 1 2 3 4 5 6 7 8 9 10 10' 9' 8' 7' 6' 5' 4' 3' 2' 1' 0' 1_2 2_2 3_2 4_2 5_2 6_2 7_2 8_2 9_2 10_2 $10_2'$ $9_2'$ $8_2'$ $7_2'$ $6_2'$ $5_2'$ $4_2'$ $3_2'$ $2_2'$ $1_2'$ $0_2'$
LOAD POINTS
ELASTIC PIER ANALYSIS

TABLE XXXVI INFLUENCE DATA FOR MOMENTS NEAR CROWN

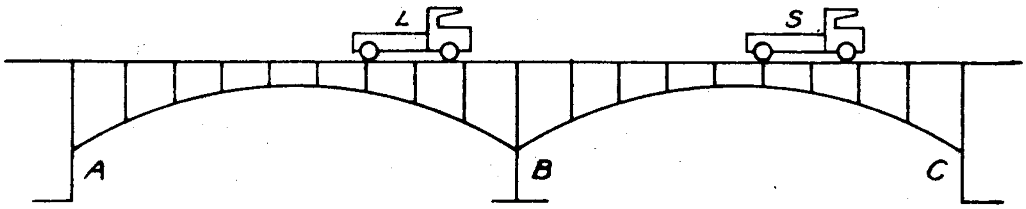
116	117	118	119	120	121	122	123	124	125	126	127
PT	M_{OA}	V_{OA}	H_{OA}	$V \times 33$	$-H \times 19.66$	$x-m$	M_{G_4}	$V \times 41$	$x-m$	$-H \times 19.66$	M'_{G_4}
1	-1.850	1.000	.0	33.000	.0	-31.150	0	41.00	-39.150	0	0
2	-4.227	.9864	.04714	32.551	- .9267	-27.450	- .0527	40.442	-35.450	- .927	- .016
3	-5.508	.9604	.1288	31.693	-2.532	-23.75	- .0970	39.376	-31.75	-2.532	- .414
4	-5.880	.9230	.2339	30.459	-4.598	-20.05	- .069	37.843	-28.05	-4.598	- .685
5	-5.536	.8756	.3519	28.895	-6.918	-16.35	.091	35.900	-24.35	-6.918	- .904
6	-4.622	.8189	.4734	27.024	-9.307	-12.65	.445	33.575	-20.65	-9.307	-1.00
7	-3.315	.7542	.5888	24.888	-11.576	- 8.95	1.047	30.922	-16.95	-11.576	- .919
8	-1.786	.6830	.6892	22.539	-13.550	- 5.25	1.953	28.003	-13.25	-13.55	- .583
9	- .1993	.6072	.7665	20.037	-15.069	- 1.55	3.219	24.895	- 9.55	-15.069	.077
G_4	.4583	.5740	.7890	18.942	-15.511	-	3.889	23.534	- 8.00	-15.511	.4813
10	1.284	.5285	.8130	17.440	-15.985		2.739	21.668	- 5.85	-15.985	1.117
10'	2.508	.4494	.8221	14.830	-16.162		1.176	18.425	- 2.15	-16.162	2.621
G'_4	3.060	.4050	.8150	13.365	-16.023		.392	16.728		-16.023	3.77
9'	3.365	.3724	.7930	12.289	-15.590		.064	15.268		-15.590	3.043
8'	3.830	.2994	.7309	9.880	-14.369		- .659	12.275		-14.369	1.736
7'	3.917	.2321	.6425	7.659	-12.631		-1.055	9.516		-12.631	.802
6'	3.663	.1718	.5350	5.669	-10.518		-1.186	7.044		-10.518	.189
5'	3.130	.1194	.4162	3.940	-8.182		-1.112	4.895		-8.182	- .157
4'	2.406	.0759	.2954	2.505	-5.807		- .896	3.112		-5.807	- .289
3'	1.592	.0414	.1815	1.366	-3.568		- .610	1.697		-3.568	- .279
2'	.802	.0167	.0845	.5511	-1.660		- .307	.685		-1.660	- .173
1'	.166	.002	.0150	.066	- .295		- .063	.008		- .295	- .120

TABLE XXXVI INFLUENCE DATA FOR MOM. NEAR CROWN CONT'D

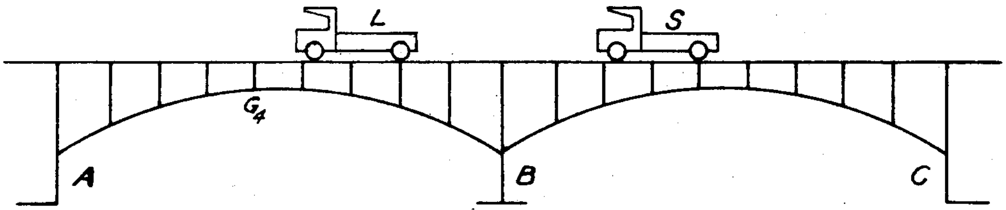
128	129	130	131	132	133	134	135	136
PT.	M_{OA}	V_{OA}	H_{OA}	$V \times 33$	$-H \times 19.66$	M_{G_4}	$V \times 41$	M'_{G_4}
1_2	0							
2_2	.0963	.000996	.00836	.03288	- .16436	-.03519	.0408	-.0273
3_2	.2574	.002655	.02231	.08761	- .4386	-.09364	.1088	-.0724
4_2	.4559	.004684	.03950	.15457	- .7766	-.16613	.1920	-.1287
5_2	.6666	.006821	.0577	.2251	-1.1344	-.2427	.2796	-.1888
6_2	.8674	.00882	.0750	.2912	-1.4745	-.3158	.3616	-.2455
7_2	1.0368	.01048	.0895	.3458	-1.7596	-.3768	.4297	-.2931
8_2	1.1571	.01159	.0997	.3825	-1.9601	-.4205	.4752	-.3278
9_2	1.2135	.01202	.1043	.3967	-2.0505	-.4403	.4928	-.3442
G_{4_2}	1.205	.0119	.1030	.3927	-2.0250	-.4273	.4879	-.3321
10_2	1.1945	.01165	.1023	.3845	-2.0112	-.43216	.4776	-.3391
$10'_2$	1.0936	.01042	.0932	.3438	-1.8323	-.3948	.4272	-.3115
G'_{4_2}	.994	.00957	.0865	.3158	-1.7005	-.3907	.3924	-.3141
$9'_2$	.9195	.00845	.0778	.2789	-1.5295	-.33117	.3464	-.2636
$8'_2$	.6940	.00596	.0580	.19678	-1.14028	-.24947	.2443	-.2020
$7'_2$	.4406	.003235	.0358	.10675	-.7038	-.1564	.13263	-.1306
$6'_2$	.1844	.000522	.01345	.01722	-.2644	-.0628	.02140	-.0586
$5'_2$	-.0478	-.001865	-.00665	-.06156	.13074	.02137	.07646	.1594
$4'_2$	-.2272	-.003619	-.0220	-.11943	.4325	.0859	.14838	.3537
$3'_2$	-.3280	-.00446	-.0304	-.14721	.59766	.1224	.18286	.4525
$2'_2$	-.3184	-.00407	-.0290	-.13434	.57014	.1174	.1668	.4185
$1'_2$	-.1662	-.00202	-.0150	-.06669	.2949	.0620	.0828	.2115



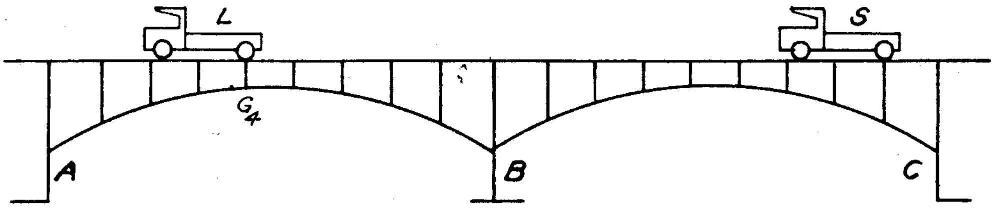
MAX NEG MOMENT AT A
(a)



MAX POS MOMENT AT A
(b)



MAX NEG MOMENT AT CROWN GIRDER G_4
(c)



MAX POS MOMENT AT CROWN GIRDER G_4
(d)

FIG 29. a,b,c,d

TABLE XXXVII MAX. MOM. & THRUST – ELASTIC PIER ANAL'S

137	138	139	140	141	142	143	144
POINT	LOAD	M	H	H COSϕ	V	V SINϕ	N
O	D.L.	-1257	+20550	+13686	+22164	+16534	+30220
	+C.L.L.	+28590	+4244	+2827	+1515	+1130	+3957
SINϕ = .746	-C.L.L.	-33312	+1268	+844	+6008	+4482	+5326
COSϕ = .666	+T	+21865	+1587	+1057			+1057
	-T	-29153	-2116	-1409			-1409
	+MOM	+49198					+35234
	-MOM	-63722					+34137
G_4	D.L.	+1446	+20550	+20441	+3239	+334	+20775
	+C.L.L.	+21225	+4832	+4806	+4073	+420	+5226
SINϕ = .103	-C.L.L.	-7857	+3575	+3556	+1115	+115	+3671
COSϕ = .9947	+T	-7272	+1587	+1578			+1578
	-T	+9696	-2116	-2105			-2105
	+MOM	+32367					+23896
	-MOM	-13683					+26024

TABLE XXXIII COMP. OF MAX STRESSES – ELASTIC PIER

145	146	147	148	149	150	160	161	162	163	164
PT	MOMENT	NORMAL	X_o INCHES	$\frac{h}{X_o}$	ηp	$\frac{N X_o}{f_c b h^2}$	K	$\frac{h-d'}{Kh}-1$	f_c LB/IN²	f_s LB/IN²
O	+49198	+35234	16.74	1.668	.143	.136	.47	.983	464	5460
	-63722	+34137	22.4	1.247	.143	.136	.42	1.22	602	8830
G_4	+32367	+23896	16.25	1.226	.156	.135	.41	1.32	602	9550
	-13683	+26024	6.30	3.170	.156	.130	.73	.31	262	975

CHAPTER VII

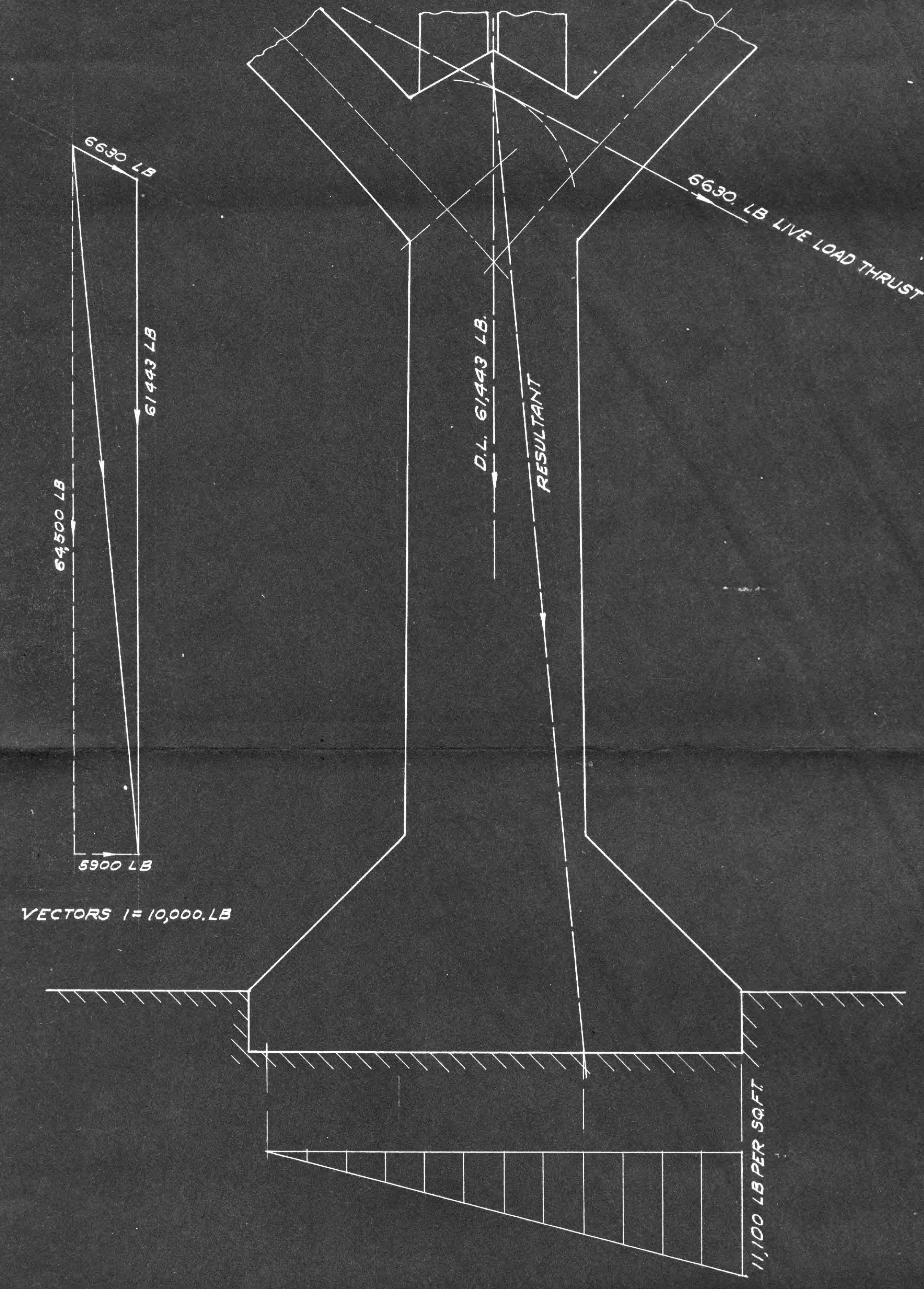
FOUNDATION AND PIER

The dead load and temperature thrusts at the center pier balance each other. A wheel load applied to the arch at girder G_4' (immediately to the right of the crown) produces the greatest unbalanced thrust on the arch and at the same time a high value of positive moment at the pier springing line. These together produce the greatest over-turning effect exerted on the pier. This is shown graphically on Fig. 30. The resulting stresses in the foundation are 11,100 pounds per square foot or slightly less than six tons per square foot, which is easily allowable.

As previously mentioned, the exact nature of the foundation conditions under the abutments was not obtained. It is proposed by the writer that the abutments be made with a sizeable shell of concrete of the usual proportions and then filled in to the outline shown on the foundation sheet with boulder concrete made perhaps by using some of the material now existing in the present piers, if this should prove upon further inspection to be satisfactory and of sufficient quantity. The reinforcing rods for the arch might be carried a considerable distance into the foundation, the stone at the top being made of smaller size.

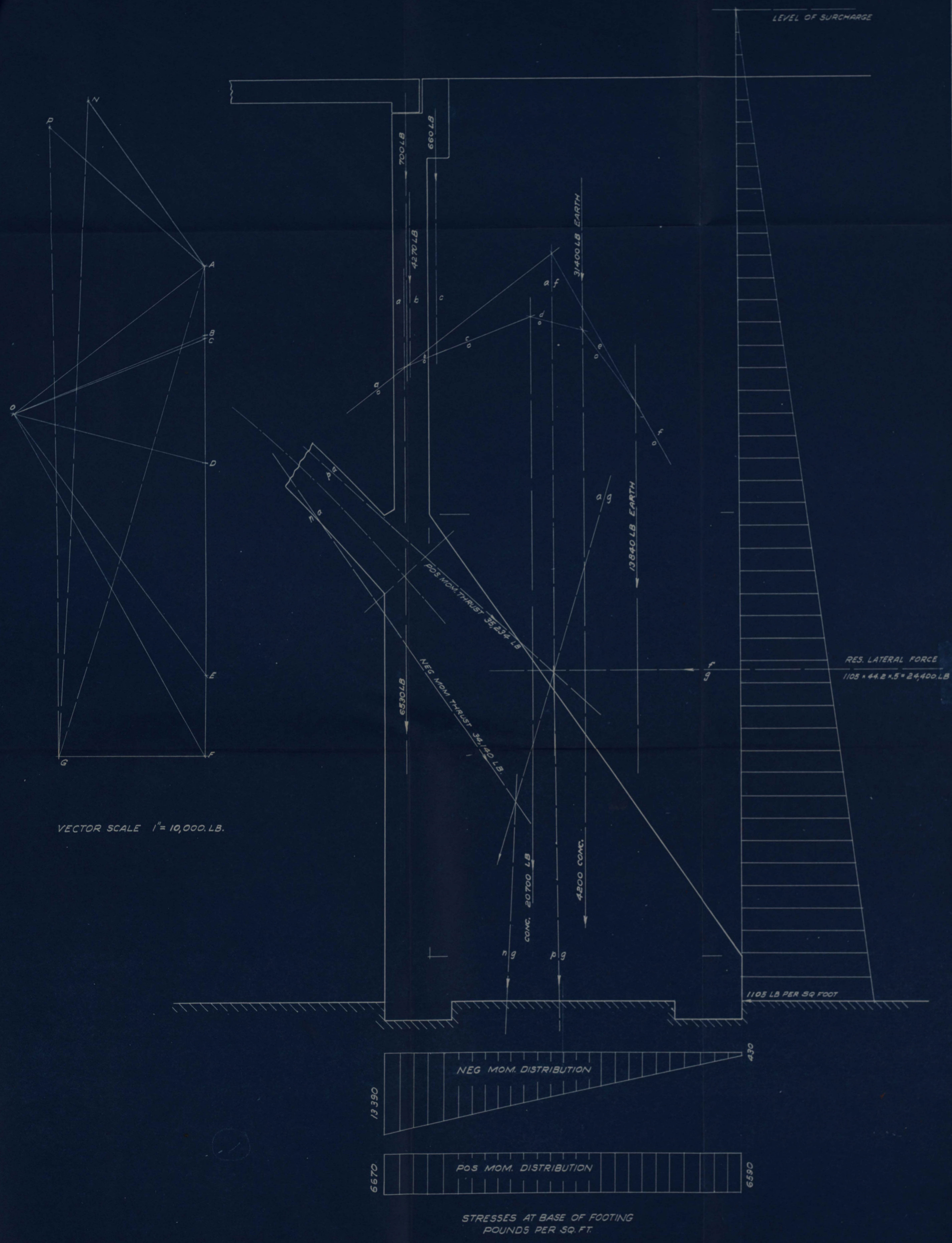
For investigating the stability of the foundation, the lateral component of the earth pressure was estimated, using a coefficient of fluidity of .25 and a surcharge of three feet. The weight of the earth filling was taken as 100 pounds per cubic foot. Using these figures, the foundation was checked for stability when acted upon by the greatest positive bending moment at the springing and also the greatest negative bending moment there. These include both dead load and temperature effects.

The results are shown graphically on Fig. 31. The greatest pressure is less than seven tons per square foot, which is allowable. The weight of the foundation and earth filling is included.



STABILITY OF CENTER PIER
BRIDGE OVER PEACHTREE CREEK
SCALE 3/8" = 1'-0"

MAY 8 1941 DRAWN BY F.M.H.



STABILITY OF FOUNDATION
BRIDGE OVER PEACHTREE CREEK
SCALE 3/8" = 1'-0"

MAY 8 1941 DRAWN BY F.M.H.

CHAPTER VIII

CONCLUSIONS

Since the primary purpose of this thesis was a comparison of the stresses in the ribs of a two-span arch on an elastic pier, some remarks on the results of the analysis are, therefore, in order.

In the first place, it must be noted that the results from this analysis are applicable only to the arch and pier arrangements having approximately the same proportions as those used here. The arch ribs used here have a rise ratio of about .26, which is probably as high as is desirable for use in ordinary construction. The center pier has a center line height of approximately seventeen feet or slightly less than the rise of the arch itself. For these proportions, it appears that if the center pier has a reasonable thickness, the increase in the stresses caused by an inclusion of the pier elasticity in the analysis is comparatively small, an increase of perhaps ten per cent. Under such circumstances, the designer might with reasonable assurance select the size of the arch ring using a fixed arch analysis and limiting the final maximum stresses to about ninety per cent of those allowable with the reasonable expectation that an elastic pier analysis would yield manimum stresses not over those allowable. It is understood, of course, that commonly accepted proportions for the arch ring thicknesses are contemplated.

Any material alteration in any of the relative values of rise ratio or of pier height as well as the inclusion of any unusual features in the design will cause a considerable change in the relative values of the stresses as given by the fixed arch analysis and the elastic pier analysis. In particular, a considerable reduction in the rise ratio or an increase in the pier height will probably cause a much greater disparity between the two values than that indicated by this analysis. In view of the fact that many arches are built with rise ratios much smaller than .26, it must be clear that it will often occur that the results of the two analyses are not nearly so close together as this comparison would indicate.

The method of analysis used here is, of course, not the only one available for such work. All arch analyses involving elastic piers are tedious; possibly this one is not the easiest. Probably it would be found that each range of span numbers in a multiple arch series has some method best suited to its analysis. Such a determination is obviously beyond the purpose of this investigation. The method used here could probably be standardized by the use of special printed forms, tables, and diagrams. When so condensed and systematized, it could be reduced to somewhat of a routine, the various operations being performed more or less automatically with but little attention to the theory. This, of course, would expedite the work of solution.

In view of the labor involved in any such analysis, it appears that a very useful service could be rendered the designer of such structures by the preparation of a set of curves which would compare the moments and thrusts given by a fixed end analysis with those which would result if pier elasticity were considered. The rise ratio of the arch and the ratio of the pier height to the rise of the arch axis could be used as variables.

With such a set of curves, the designer might select the proportions of the arch desired, basing his selection on the comparatively easy fixed-end analysis, knowing in advance that such analysis would give stresses that were too low but by a known amount. In this way, several trials could be made, and a satisfactory set of arch proportions could be arrived at by very little more labor than would be required in the case of a single-span fixed arch. A final check using the elastic pier analysis would, of c ourse, be made, but the results should be nearly predictable in advance.

Probably the compilation of such a set of curves, involving as it must such a vast amount of labor, could be undertaken only with the assistance of a staff of computers.

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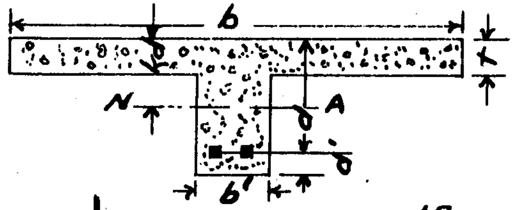
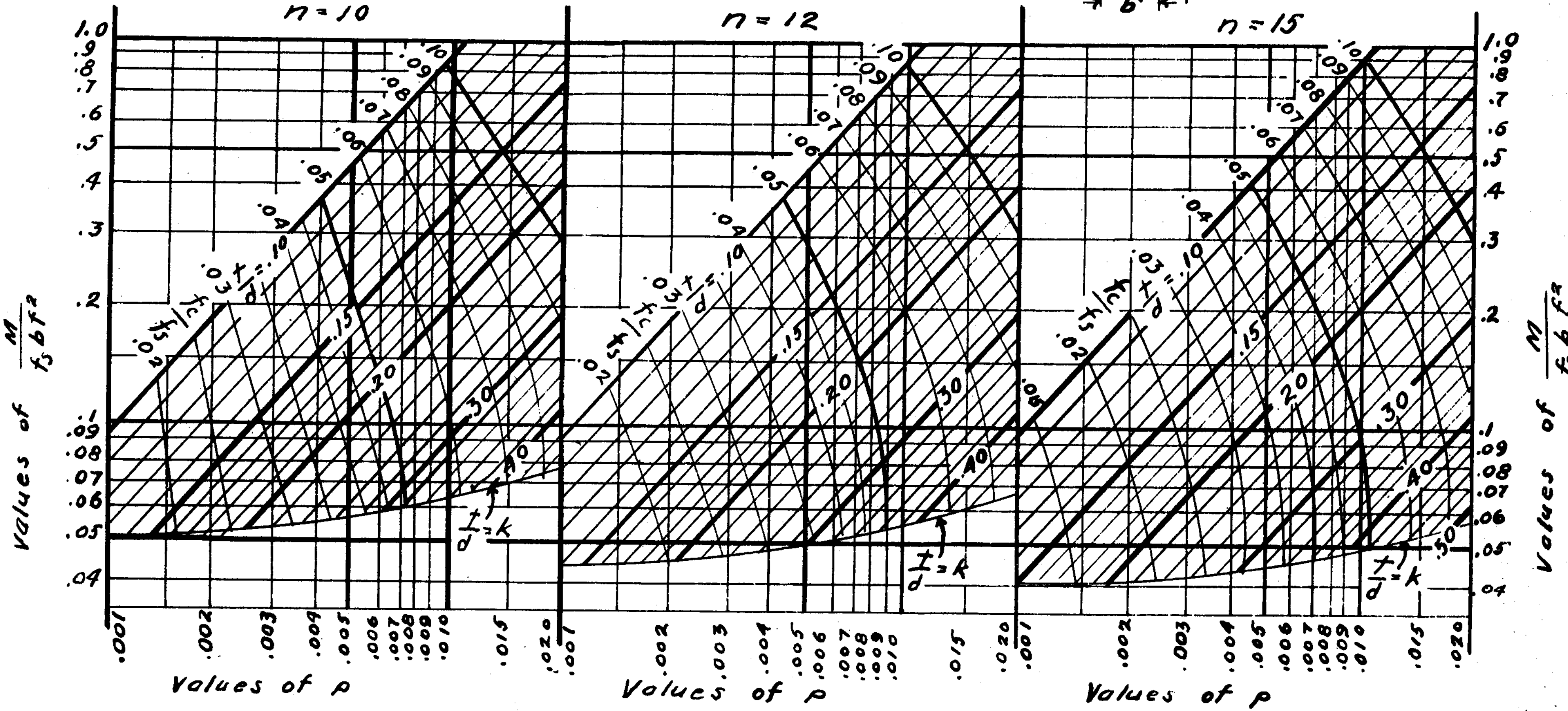
APPENDIX

$$\frac{t}{d} = \frac{R_2 - \sqrt{R_2^2 - 12 f_c R_1}}{2 R_1} \qquad p = \left(\frac{t}{d}\right)\frac{f_c}{f_s} - \left(\frac{t}{d}\right)^2 \frac{1}{2}\left(\frac{f_c}{f_s} + \frac{1}{n}\right)$$

Where $R_1 = f_c + \frac{f_s}{n}$ $\quad R_2 = \frac{M}{bt^2} + f_c + \frac{f_s}{2n}$



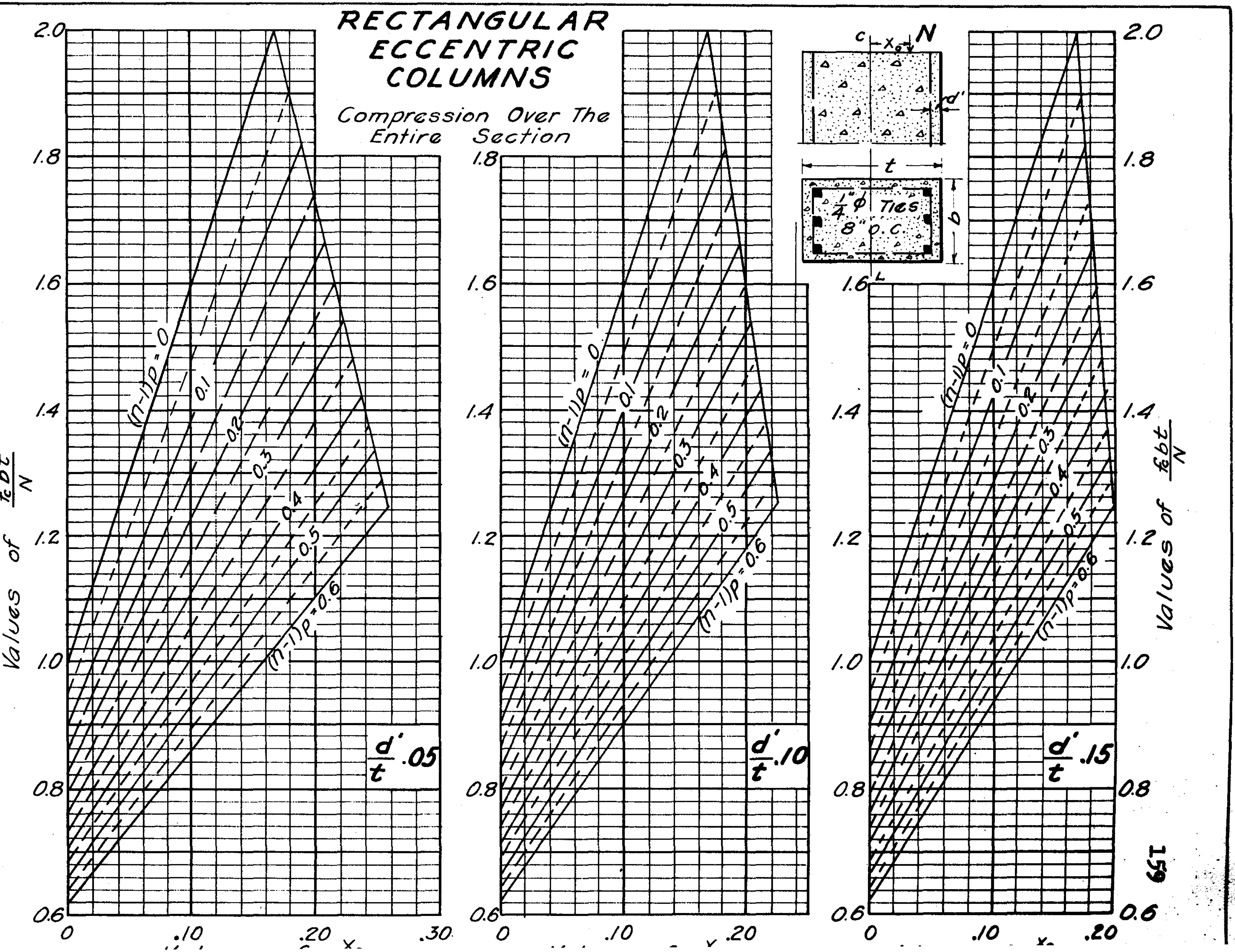


"T" BEAMS.

DIAGRAM 2.

RECTANGULAR ECCENTRIC COLUMNS

Compression Over The Entire Section



RECTANGULAR ECCENTRIC COLUMNS

Tension over Part of the Section

$f_s = nf_c\left[\frac{1}{k} - 1\right]$